

The coskewness puzzle

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Abstract

We propose a novel approach to testing non-linear stochastic discount factor (SDF) specifications that arise in rational representative investor models. Our approach does not require overly-restrictive assumptions about the shape of investors' preferences, typically imposed by the extant literature, and is based instead on restrictions that rule out "good deals", i.e. arbitrage opportunities as well as unduly large Sharpe ratios. We apply this framework to test the empirical admissibility of 3 and 4-moment versions of the CAPM. We find that, while coskewness and cokurtosis risk help price a number of stock strategies and portfolios, including static strategies based on a fine industry-level diversification, momentum strategies and portfolios managed on the basis of available information, the CAPM and its 3 and 4-moment versions cannot provide an exhaustive account of observed asset returns.

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We propose a novel approach to testing non-linear stochastic discount factor (SDF) specifications that arise in rational representative investor models. Our approach does not require overly-restrictive assumptions about the shape of investors' preferences, typically imposed by the extant literature, and is based instead on restrictions that rule out "good deals", i.e. arbitrage opportunities as well as unduly large Sharpe ratios. We apply this framework to test the empirical admissibility of 3 and 4-moment versions of the CAPM. We find that, while coskewness and cokurtosis risk help price a number of stock strategies and portfolios, including static strategies based on a fine industry-level diversification, momentum strategies and portfolios managed on the basis of available information, the CAPM and its 3 and 4-moment versions cannot provide an exhaustive account of observed asset returns.

1. Introduction

In a literature that spans almost thirty years, starting with the seminal studies of Kraus and Litzenberger (1976) and Friend and Westerfield (1980), considerable evidence has accumulated suggesting that differences in average returns across assets are related to systematic third and fourth moments of the return distribution. Harvey and Siddique (2000) show that coskewness with a market portfolio proxy is important in explaining the cross-section of returns on US stocks and commands on average a risk premium of 3.6 percent per annum. Dittmar (2002) finds that cokurtosis, as well as coskewness, is priced in the cross-section of industry-sorted stock portfolios. Smith (2007) confirms that the price of coskewness risk is empirically large, especially at times when market skewness is negative. Chunchinda et al. (1997) and, more recently, You and Daigler (2010) argue that investors trade expected return for skewness in forming portfolios of International stocks. Moreno and Rodriguez (2009) analyze the role of coskewness in mutual fund performance evaluation and find evidence that adding a coskewness factor is economically and statistically significant.

Attempts to explain this body of evidence have largely relied on higher moment versions of the Capital Asset Pricing Model (CAPM). Kraus and Litzenberger (1976) put forth a 3-moment CAPM (henceforth, 3M-CAPM), that can be seen as a direct extension of the static 2-moment CAPM (2M-CAPM) originally developed by Sharpe (1964) and Lintner (1965). This model is later generalized, in a conditional setting, by Dittmar (2002), who also considers a 4-moment extension (4M-CAPM), and Smith (2007). A crucial prediction of the CAPM, including its higher moment versions, is that the market portfolio is efficient. In this model, the inter-temporal marginal rate of substitution (henceforth, IMRS) of a rational, expected utility-maximizing representative investor can be well approximated as a polynomial in the return on the market portfolio and it is a valid choice for the stochastic discount factor (henceforth, SDF) that prices all assets.

Consistent with the 3M and 4M-CAPM, Dittmar (2002) finds that SDF specifications quadratic and cubic in the market return provide a much better fit to the observed cross-section of stock returns than linear models. This author also emphasises, however, that the best fit obtains with a U-shaped SDF, whereas the superior performance of the 3M and 4M-CAPM is significantly reduced when the SDF is restricted to be decreasing in the market return. Post, Levy and van Vliet (2008) report similar findings and reach similar conclusions. If we interpret the estimated SDF as the representative investor's IMRS, this evidence implies that marginal utility of wealth increases above a threshold and thus that the utility function takes an 'inverse S' shape. Such a shape, in turn, implies risk-seeking over a range of sample wealth.

It is not uncommon, in the extant literature, to encounter studies that find evidence of risk-seeking and specifications of investors' preferences that admit this type of behaviour. Psychologists, led by Kahneman and Tversky (1979), find experimental evidence of local risk seeking. Friedman and Savage (1948) and Markowitz (1952) suggest that the willingness to purchase both insurance and lottery tickets implies that marginal utility is increasing over a range of investors' wealth. Golec and Tamarkin (1998) and Garrett and Sobel (1999) provide evidence that risk-averse individuals take actuarially unfair "long shot" gambles, i.e. low-probability but high-variance bets, and argue that this behaviour can be rationalized by postulating skewness preference and an inverse-S shaped utility function. Post and van Vliet (2006) and Post, Levy and Van Vliet (2008) argue, however, that non-concave utility is problematic from the viewpoint of the 3M-CAPM. In essence, they point out that, if the representative investor's utility function is not concave, the market portfolio is not guaranteed to maximize her expected utility function and the 3M-CAPM does not necessarily hold even if the estimated SDF is quadratic in the return on such portfolio. Their remark extends in a straightforward manner to models of the SDF cubic in market wealth, implying a similar difficulty in interpreting the explanatory power of such models as evidence in favour of the 4M-CAPM.

The problematic shape of the representative investor's utility function, implied by unrestricted estimates of the 3M and 4M-CAPM, represents a puzzling conundrum. On the one hand, Dittmar's (2002) findings and the critique put forth by Post, Levy and Van Vliet (2008) suggest that omitting appropriate restrictions on the shape of the candidate SDF might lead to over-fitting the cross-section of the test asset returns and thus to spurious estimates of the parameters of the 3M and 4M-CAPM. On the other

hand, concavity of utility is a sufficient but not necessary condition for the market portfolio to maximize expected utility. This implies that it is formally impossible to make any conclusive inference on the empirical validity of higher moment versions of the CAPM if the latter are rejected when this condition is imposed in estimation, as in the tests performed by Dittmar (2002) and Post, Levy and Van Vliet (2008). Such tests, in fact, while suggestive, amount to tests of the joint hypothesis that the higher moment CAPM holds and utility is concave.

To mitigate this difficulty, we pursue an alternative approach. We only assume that marginal utility is positive and the representative investor's relative risk aversion (RRA) does not exceed an upper bound motivated by survey and experimental evidence (secondary data) on economic agents' risk aversion, as well as by introspection. These restrictions, and especially the bound on RRA, have the effect of limiting the volatility of the estimated SDF and, together, they rule out both arbitrage opportunities and unduly high Sharpe ratios (henceforth, SR). Imposing such restrictions boils down to limiting the admissible curvature of the representative investor's utility function and, ultimately, mitigates the danger of over-fitting the cross-section of the test asset returns. Because this approach does not rely on overly-restrictive assumptions about the shape of utility, it permits a more direct appraisal of the empirical admissibility of higher-moments versions of the CAPM than the approach followed by Dittmar (2002) and Post, Levy and Van Vliet (2008).

Our empirical results paint a complex picture. On the one hand, somewhat surprisingly, we find that coskewness and cokurtosis risk matter only for the pricing of strategies characterized by relatively high SRs, such as dynamic portfolios managed on the basis

of available conditioning information or, at least until the late 90s, payoffs spanned by a fine industry-level diversification, as well as momentum strategies. On the other hand, our results conclusively demonstrate that, as suspected by Post and van Vliet (2006), the CAPM and its higher-moment versions cannot provide an exhaustive description of average stock returns. This is also true of specifications, akin to those estimated by Dittmar (2002), consistent with 3 and 4-moment conditional versions of this model. We conclude that, while preferences towards higher moments are important in explaining average returns, especially on certain strategies, the CAPM and its higher-moment versions provide at best a partial account of the stock price determination mechanism.

In the next Section, we discuss the problem of bounding from above SDF volatility in the context of the higher-moment CAPM. In Section 3, we outline the estimation methodology that underlies our tests. In Section 4, we describe our dataset. In Section 5, we present our empirical results for the case of unconditional models. In Section 6, to facilitate comparisons with the extant literature, we report estimates of the unconditional models obtained using different sample periods and test asset payoffs. In Section 7, we report our empirical results for the case of conditional models. In Section 8, we discuss the implications of our findings and offer our conclusions.

2. The CAPM, risk aversion and SDF volatility

The economy SDF is defined as the random variable m_{t+1} that satisfies the following condition for all traded payoffs x_{t+1} and payoff prices p_t :

$$p_t = E_t(m_{t+1}x_{t+1}) \tag{1}$$

Here, the expectation is taken conditional on the available information set. A well known theorem, credited to Harrison and Kreps (1979), implies that, given free portfolio formation and the law of one price, at least one such variable exists and, under the additional assumption of no arbitrage, it is positive. We specify the candidate SDF as the following third order polynomial defined over the market portfolio return $R_{m,t+1}$:

$$m_{t+1} = a_t + b_{1,t}R_{m,t+1} + b_{2,t}R_{m,t+1}^2 + b_{3,t}R_{m,t+1}^3 \quad (2)$$

The specification in (2) can be interpreted as a factor model with factors given by the first three integer powers of the return on the market portfolio of risky assets. In view of the covariance-pricing representation of the implications of (1) and (2) outlined in Appendix A, $-b_{1,t}$, $-b_{2,t}$ and $-b_{3,t}$ can be seen as the prices of market covariance, coskewness and cokurtosis risk, respectively. It is well known that the CAPM imposes a number of restrictions on (2). In this model, the market portfolio is the inter-temporal optimal allocation that maximizes the representative investor's multi-period expected utility, subject to the economy's inter-temporal budget constraint. Denoting by $U(W_{t+1})$ the investor's time-separable utility function of wealth and by $U'(W_{m,t+1})$ its first derivative, the first order condition (FOC) for the optimum is that, for every payoff x_{t+1} , the following equality holds:

$$U'(W_{m,t})p_t = \mathcal{E}_t[U'(W_{m,t+1})x_{t+1}] \quad (3)$$

Here, $W_{m,t+1}$ denotes market wealth whereas γ , with $0 < \gamma < 1$, captures impatience¹.

Dividing both sides of (3) by $U'(W_{m,t})$, we obtain the following well known pricing result:

$$p_{t+1} = \frac{\gamma E_t [U'(W_{m,t+1})x_{t+1}]}{U'(W_{m,t})} = E_t [\varphi_{t+1}(W_{m,t+1})x_{t+1}] \quad (4)$$

Here, $\varphi_{t+1} \equiv \gamma \frac{U'(W_{m,t+1})}{U'(W_{m,t})}$ is the representative investor's IMRS. A comparison between

(1) and (4) demonstrates that, under the CAPM, the IMRS is an admissible SDF. Taking a third order Taylor expansion around the initial wealth level $W_{m,t}$, the IMRS can be approximated as follows:

$$\varphi_{t+1} \cong \gamma + \gamma \frac{U''(W_{m,t})}{U'(W_{m,t})} W_{m,t} R_{m,t+1} + \frac{1}{2} \gamma \frac{U'''(W_{m,t})}{U'(W_{m,t})} W_{m,t}^2 R_{m,t+1}^2 + \frac{1}{6} \gamma \frac{U^{(4)}(W_{m,t})}{U'(W_{m,t})} W_{m,t}^3 R_{m,t+1}^3 \quad (5)$$

By identification between (2) and (5), we have the following restrictions on the parameters of the candidate SDF:

$$a_t = \gamma \quad b_{i,t} \equiv \gamma \frac{U^{(i)}(W_{m,t})}{U'(W_{m,t})} W_{m,t} \quad i \in [1, 2, 3, 4] \quad (6)$$

Here, $U^{(i)}$ denotes the i -th derivative of the utility function $U(W_{m,t})$. Arrow (1971) argues that desirable properties of this function are non satiation (NS), risk aversion

¹ The coefficient γ is often called the subjective inter-temporal rate of substitution.

(RA) and non-increasing absolute risk aversion (NIARA). NS implies $U' > 0$, RA implies decreasing MU, i.e. $U'' < 0$, whereas NIARA implies that the rate of decrease of MU does not increase in wealth and thus $U''' \geq 0$. Hence, NIARA implies aversion to negative skewness of the distribution of the return on wealth. Kimball (1993) adds non-increasing absolute prudence (NIAP) to the set of desirable properties of a 'well behaved' utility function. Under NIARA, NIAP implies $U'''' \geq 0$ and thus aversion to kurtosis. Together, NS, RA, NIARA and NIAP yield standard risk aversion, as defined by Kimball (1993). Preferences of rational expected utility maximizers will necessarily display the first of these properties, namely NS. This implies a no-arbitrage (NA) positivity restriction on the sign of the admissible SDFs, and hence $m_{t+1} > 0$, to avoid assigning a zero or negative price to strictly positive payoffs and thus to rule out unexploited arbitrage opportunities. NIAP, NIARA and RA rule out counter-intuitive behaviour and can be used to further restrict m_{t+1} , but they are not necessary conditions for the CAPM. When $U' > 0$, and thus under NS, NIAP implies $b_3 \leq 0$. Under NS, a necessary condition for $U''' \geq 0$, and thus for NIARA, is $b_2 \geq 0$. Finally, a necessary condition for $U'' < 0$, and thus for RA, is $b_1 < 0$. The latter condition guarantees local risk aversion, a milder requirement than concave utility (i.e., global risk aversion).

The key prediction of the CAPM is that the market portfolio is efficient. This requires that the market portfolio maximizes the representative investor's expected utility, subject to the inter-temporal budget constraint. That is, the market portfolio must represent the solution to a constrained maximization problem. The equalities in (3) represent FOCs of this problem. A necessary and sufficient condition for their solution to coincide with the solution to the representative investor's problem is that the

expectation of the utility function (the maximand) be quasi-concave in wealth. Deriving the implications of this condition for the parameters of (2) is, however, extremely complex. This difficulty explains why, in tests of the higher-moment CAPM, researchers usually impose global concavity of utility and thus require that its second derivative U'' be negative, by either using a power specification or suitably restricting the parameters of a polynomial expansion of the utility function, as in Dittmar (2002). This condition is relatively easy to enforce and it is sufficient to guarantee that the expectation of utility be quasi-concave in wealth, but it is not necessary for this to be the case. Therefore, if the aim is to test higher moment versions of the CAPM, such condition is too restrictive.

We do not wish to impose the overly restrictive assumption that utility is concave, nor we are able to directly impose the requirement that its expectation be quasi-concave. We do wish, however, to restrict the curvature of the utility function in a meaningful fashion. To achieve this aim, we exploit the link between the curvature and volatility of the representative investor's IMRS. As a first step, we obtain an expression for the variance of the IMRS by applying the conditional variance operator to both sides of (5). In doing so, we neglect all terms in the Taylor expansion of the IMRS that contain second and higher order powers of the market return. This yields the following locally valid approximation of the IMRS conditional variance:

$$\sigma_i^2(\varphi_{t+1}) \cong \gamma^2 \sigma_i^2 \left[\frac{U''(W_{m,t})}{U'(W_{m,t})} W_{m,t} R_{m,t+1} \right] \cong \gamma^2 \sigma_i^2 (RRA_{m,t} R_{m,t+1}) \quad (7)$$

The above result illustrates the relation between RRA and the volatility of the IMRS in a neighbourhood of an initial wealth value $W_{m,t}$. Under an upper bound on the (absolute value of) RRA, i.e. letting $|RRA_{m,t}| \leq RRA_V$, we have the following inequality:

$$\sigma_t^2(RRA_{m,t}R_{m,t+1}) \leq \sigma_t^2(RRA_V R_{m,t+1}) = RRA_V^2 \sigma_t^2(R_{m,t+1}) \quad (8)$$

Using (8) to bound from above the right-hand side of (7), and recalling that $0 < \gamma \leq 1$, we obtain the following upper bound to the volatility of the IMRS:

$$\sigma_t^2(\varphi_{t+1}) \leq \gamma^2 RRA_V^2 \sigma_t^2(R_{m,t+1}) \leq RRA_V^2 \sigma_t^2(R_{m,t+1}) \quad (9)$$

This inequality, ultimately a restriction on the curvature of utility, implies that the volatility of the IMRS is bounded from above by a quantity that depends on market volatility and a bound on the admissible values of RRA. It should be noted that, while (7) only holds in a neighbourhood of the initial wealth value $W_{m,t}$, the upper bound in (9) holds in the neighbourhood of every value of wealth as long as RRA, itself a function of wealth, evaluates to a quantity that is within the RRA bound. This makes the bound relevant for restricting the volatility of the IMRS of investors who exhibit non constant RRA and thus care about moments of third and higher orders. Therefore, (9) can be seen as a generalization to a possibly non mean-variance world of a result already formulated by Ross' (2005) under the assumption of either log-normally distributed returns or exponential utility. Importantly, Ross (2005) also demonstrates² that the volatility of the representative investor's IMRS provides a "no good deal" upper

² See Proposition 1 in Ross (2005).

bound to the economy maximal SR or, equivalently, to the volatility of the minimum-variance SDF that prices all assets traded in the economy, i.e. $\sigma_i^2(m_{t+1}) \leq \sigma_i^2(\varphi_{t+1})$.

Thus, using (9), we obtain the following bound on the variance of m_{t+1} :

$$\sigma_i^2(m_{t+1}) \leq \sigma_i^2(\varphi_{t+1}) \leq RRA_V^2 \sigma_i^2(R_{m,t+1}) \quad (10)$$

Taking unconditional expectations of both sides of (10) gives the following upper bound on the unconditional variance of the candidate SDF:

$$\sigma^2(m_{t+1}) \leq RRA_V^2 \sigma^2(R_{m,t+1}) \quad (11)$$

To give empirical content to this inequality, one needs to identify a suitable value for the RRA bound. To this end, we follow Ross' (2005) advice and seek guidance from survey and experimental evidence on investors' RRA provided by the extant literature. Ross (2005) suggests imposing an upper bound of 5 on the RRA of the marginal investor, i.e. $RRA_V = 5$. Among the motivations advanced by Ross (2005) to do so, the one that most easily applies to a world with possibly non-normally distributed returns and non quadratic utility is the simple observation that RRA higher than 5 would imply that the investor is willing to pay more than 10 percent per annum to avoid a 20 percent volatility of his wealth (i.e., about the unconditional volatility of the S&P in the last 75 years), which seems a rather large amount. Similar indications can be drawn from the analysis offered by Meyer and Meyer (2005), who provide a comprehensive re-evaluation of the hitherto scattered empirical evidence on economic agents' risk aversion. They show that RRA estimates reported by the extant literature are less

heterogeneous and extreme if one takes into account measurement issues and the outcome variable with respect to which each study defines risk aversion. Using returns on stock investments as the outcome variable, calculations by Meyer and Meyer (2005) show that the RRA coefficient in the classical Friend and Blume's (1975) study of household asset allocation choices ranges between 6.4 and 2.0, and decreases in investors' wealth. Using returns on the investors' overall wealth, including real estate and a measure of human capital, the RRA estimate ranges between 3.0 and 2.4. The same calculations show³ that the RRA implied by the findings of Barsky et al. (1997) ranges between 0.8 and 1.6.

These considerations allow us to identify a plausible upper bound on RRA. To be able to compute the SDF volatility upper bound in (11), all that remains to be done is to obtain an estimate of the market portfolio volatility, or at least an upper bound on such a quantity. To this end, we note that, even though the market portfolio of risky assets includes both a traded and a non traded portion, the former is likely the most volatile. Thus, we use the S&P500 index to proxy for the traded portion and neglect the non-traded one, which is hard to measure. The sample estimate of the S&P500 index volatility over the period 1965-2005 is about 15.5 percent per annum. Therefore, using (11), the SDF volatility bound for this period is about 78 percent per annum if, as suggested by Ross (2005), the RRA upper bound is set to $RRA_V = 5.0$ and about 99 percent per annum if the RRA upper bound is set to match the RRA of the most risk averse households' cohort in Friend and Blume's (1975) study, i.e. $RRA_V = 6.4$.

³ Meyer and Meyer (2005) calculate somewhat higher values based on estimates provided by studies of the equity premium puzzle. Since these estimates are backed out parametrically from estimates of a particular asset pricing model, often based on a narrow definition of the market portfolio, they are of no interest for the purpose of computing the SDF volatility bound. Moreover, their use would imply a circular argument.

3. Factor model estimation under SDF sign and volatility restrictions

Given a set of n basis test asset payoffs x_{t+1} , a NA restriction and an upper bound on the volatility of the candidate SDF, the latter can be estimated by solving the following problem:

$$\begin{aligned} \min_{\{m\}} \quad & g_t' W_{n \times n} g_t \\ \text{s.t.} \quad & m_{t+1} \geq 0, \quad \sigma_t^2(m_{t+1}) \leq A_t \end{aligned} \tag{12}$$

with

$$g_t = E_t(m_{t+1} x_{t+1}) - p_t(x_{t+1}) \tag{13}$$

The elements of the vector g_t correspond to the moment conditions implied by the restriction in (1) and can be interpreted as pricing errors, while W is a suitable $n \times n$ weighting matrix. The efficient choice for the latter is Hansen's (1982) optimal weighting matrix but in this study, following Dittmar (2002), we mainly use Hansen and Jagannathan (1997) second moment matrix because it does not reward spurious variability of the candidate SDF. When using this weighting matrix, the minimized pricing error metric in (12) becomes T times the square of Hansen and Jagannathan (1997) distance, where T denotes the length of the sample period. To make inferences about the empirical admissibility of the candidate SDF, we use the asymptotic distribution of this statistic, under the null of zero pricing errors, provided by Jagannathan and Wang (1996). To identify the mean of the SDF, as done by Dittmar (2002) and recommended by Dahlquist and Soderlind (1999) and Farnsworth et al.

(2002), we include the risk-free asset amongst the basis test asset payoffs x_{t+1} . In estimation, under the usual ergodicity assumptions, we use sample averages $E_T(\cdot)$ instead of unconditional means and we expand the set of orthogonality conditions by imposing the pricing errors to be unpredictable using information carried by a vector of instruments z_t :

$$\min_{\{m\}} g_T' W_{nk \times nk} g_T \quad (14)$$

$$s.t. \quad m_{t+1} \geq 0, \quad \sigma_T^2(m_{t+1}) \leq A$$

$$g_T = E_T \left[(m_{t+1} x_{t+1} - p_t) \otimes z_t \right] \quad (15)$$

Here, z_t is a vector of k conditioning variables. This yields a total of $N = n \times k$ orthogonality conditions for the pricing of as many test assets with payoffs represented by the cross-products $x_{t+1} \otimes z_t$. The latter can be seen as payoffs of dynamic portfolios made up of basis assets having payoffs x_{t+1} and managed using information carried by the conditioning variables z_t . The candidate SDF m_{t+1} is the polynomial described by (2). We call the unrestricted version of the latter the (conditional) cubic market factor model, and denote it by the shorthand notation CMFM. Letting $b_{2,t}$ equal zero yields a quadratic market factor model (QMFM). Setting both $b_{2,t}$ and $b_{3,t}$ equal to zero yields a linear market factor model (LMFM). Given the seminal role played by Dittmar's (2002) study in the literature on tests of the higher moment-CAPM under preference restrictions, we adopt an empirical specification of the candidate SDF that is as close as possible to the one used by this author. We thus model the market portfolio as a linear

combination of a value-weighted portfolio of traded assets, with rate of return denoted by $R_{vw,t+1}$, and human capital, with rate of return $R_{l,t+1}$. That is, we let:

$$R_{m,t+1} = \theta_t R_{vw,t+1} + (1 - \theta_t) R_{l,t+1} \quad (16)$$

Here, $0 \leq \theta_t \leq 1$ represents the fraction of market wealth allocated to traded financial assets. Thus, using (16) in (2), we have the following empirical specification of the candidate SDF:

$$m_{t+1} = a_{0,t} + b_{vw,1,t} R_{vw,t+1} + b_{vw,2,t} R_{vw,t+1}^2 + b_{vw,3,t} R_{vw,t+1}^3 + b_{l,1,t} R_{l,t+1} + b_{l,2,t} R_{l,t+1}^2 + b_{l,3,t} R_{l,t+1}^3 \quad (17)$$

Here, $b_{j,t} = \frac{b_{vw,j,t} R_{vw,t+1}^j + b_{l,j,t} R_{l,t+1}^j}{\theta_t R_{vw,t+1}^j + (1 - \theta_t) R_{l,t+1}^j}$, with $j = [1, 2, 3]$. This specification is the same as

the one used by Dittmar (2002). We note, however, that the weights θ_t and $1 - \theta_t$ are time-varying. For this reason, imposing that the candidate SDF in (17) be decreasing in $R_{vw,t+1}$ and $R_{l,t+1}$, as done by Dittmar (2002), is not the same as imposing it to be decreasing in $R_{m,t+1}$. In fact, the former requirement is much more restrictive than the latter and it is overly restrictive if the aim is to impose concavity of the utility function, which only requires that the candidate SDF be decreasing in the overall market portfolio wealth rather than in each one of its components⁴. These considerations provide further motivation for imposing positivity and volatility restrictions on the estimated SDF in place of global concavity in tests of the higher moment CAPM. Finally, following again

⁴ Admittedly, imposing that the estimated SDF be decreasing in $R_{m,t+1}$, rather than in $R_{vw,t+1}$ and $R_{l,t+1}$ separately, can be quite challenging because of obvious programming difficulties and econometric

Dittmar (2002), we model conditional time-variation in the shape of the utility function by specifying the parameters of the candidate SDF as linear functions of the set of conditioning information variables z_t , i.e. we let,

$$a_t = \eta' z_t \quad b_{vw,j,t} = \phi_{vw,j}' z_t \quad b_{l,j,t} = \phi_{l,j}' z_t \quad (18)$$

Here, recalling that z_t is a vector of k conditioning variables, $\phi_{vw,j}$ and $\phi_{l,j}$, with $j = [1, 2, 3]$, are conformable coefficients vectors.

4. Empirical specification details and data

We use monthly data, from 1963 to 2005, on the rate of returns on an overall value-weighted portfolio of stocks included in the Centre for Research on Security Prices (CRSP) database and on portfolios of such stocks sorted into 17 and 30 industries as well as 10 momentum deciles⁵. As customary, the value-weighted portfolio represents the proxy for the return $R_{vw,t+1}$ on the traded portion of the market portfolio of risky assets. The returns on the 17 and 30 industry-sorted stock portfolios represent payoff spaces similar to those spanned by the returns on the 20 and 27 industry-sorted portfolios used by Dittmar (2002) and Harvey and Siddique (2000), respectively. The 10 momentum portfolios are also similar to those used by the latter authors. We use NIPA data to construct, as in Dittmar (2002), a proxy for the return on human capital $R_{l,t+1}$ and we employ the yield of the 1-month US Government Treasury Bill as a proxy for the risk-free rate. We use the rates of return on the test asset payoffs to compute

software limitations as well as because the weights with which traded and non traded wealth enter the market portfolio are difficult to observe and measured with error.

gross returns (i.e., one plus the rate of return or the yield). The conditioning information variables z_t are the unit series and variables drawn from a set that includes the excess-return on the market portfolio proxy $r_{m,t}$, its dividend-yield dy_t , the yield spread ys_t of the 3-month T-Bill in excess of the 1-month T-Bill, the return tb_t on the T-Bill closest to one-month maturity. These variables are known to predict market-wide returns, as noted by Dittmar (2002) and references therein.

Figure 1 plots the Hansen and Jagannahan (1991) mean-variance frontier of the SDFs that price by construction the payoff space for the case when the payoffs x_{t+1} are the gross returns either on one of the two sets of industry portfolios or on the 10 momentum portfolios, augmented by the gross return on the risk-free asset proxy. The figure also plots the SDF mean-variance frontier for the case when the payoffs are the cross-products $x_{t+1} \otimes z_t$ between the gross returns on the 17 industry-sorted portfolios, augmented by the gross return on the risk-free asset proxy, and the conditioning variables z_t , represented by the unit series, $r_{m,t}$, dy_t , ys_t and tb_t . Panel A of the Figure plots the frontiers for the sub-sample period 1965-1993, which ends at the same time as the sample period considered by Harvey and Siddique (2000), while Panel B plots the frontiers for our benchmark sample period 1965-2005. By construction, the 30 industry-sorted portfolios span all strategies spanned by the 17 industry-sorted ones, but not the other way round. As a result, the frontier for the former set of portfolios lies above the frontier for the latter. Similarly, the managed portfolios span all strategies spanned by the static portfolios, but not the other way round, which is reflected in a tighter Hansen and Jagannahan (1991) bound for the former and thus in a higher frontier. One

⁵ We thank K. French for making this data publicly available for download.

interesting feature of all these frontiers is that they are noticeably lower in the longer sample period.

5. Unconditional estimates

We first estimated unconditional versions of (17), setting z_t equal to the unit series and using gross returns as test asset payoffs. The price of these payoffs, by construction, equals one. In (15), we therefore set $p_t = 1$. In Panel A of Table 1, we summarize the unconstrained estimates of the LMF, QMF and CMF. The panel reports the point estimates of the coefficients of the candidate SDFs, along with the p -values of the corresponding t -statistics computed using heteroskedasticity and autocorrelation adjusted (HAC) Newey and West (1987) standard errors, the Hansen and Jagannathan (1997) distance and the associated p -value, based on the asymptotic distribution under the null of zero pricing errors provided by Jagannathan and Wang (1996), as well as the annualized volatility of the estimated SDF and the implied RRA bound. The sample period is 1965-2005.

The estimated b_2 coefficients are statistically insignificant for all models. While the non-linear models are not rejected, neither are the linear ones. As shown in Panel B of the Table, imposing the positivity constraint on the estimated SDF does not significantly diminish the explanatory power of the models under consideration and the restriction is in many cases not binding. In Table 2, we report estimates of the candidate SDFs obtained under additional restrictions. To enforce RA and NIARA in estimation, we imposed $(-1)^j b_{i,j,t} > 0$, with $i \in [vw, l]$ and $j \in [1, 2, 3]$. We also restricted the estimated SDF to satisfy a strict positivity constraint and imposed an upper bound on its

volatility, as specified by (14). The volatility bound was set to 78 percent per annum, corresponding to a RRA upper bound of 5. Again, the Hansen and Jagannathan (1997) test fails to reject any of the models under consideration. Overall, these results suggest that third and fourth systematic moments play at most a limited role in the pricing of the test asset payoffs used in estimation, i.e. the gross returns on the 17 industry-sorted portfolios and the risk-free asset.

6. The role of coskewness risk over time and strategies

On balance, our findings thus far suggest that the price of coskewness (and cokurtosis) risk does not greatly matter in the pricing of unconditional (i.e., static) portfolios of stocks. This is consistent with the results reported by Friend and Westfield (1980) and Fang and Lai (1997) but is in sharp contrast with the findings of other authors, most notably Harvey and Siddique (2000). To double-check on these seemingly conflicting results, we estimate the QMFM under alternative choices for the weighting matrix of the moment conditions, including the identity matrix and the optimal weighting matrix, and using alternative sets of test asset payoffs and sample periods.

To save space, we report in Table 3 only the estimation results obtained using a sample counterpart of Hansen's (1982) optimal weighting matrix, based on a continuously updating (CUE) estimate of the spectral density matrix, and a set of test asset payoffs represented by excess-returns (in place of gross returns) on the 17 and 30 industry-sorted portfolios over the sample period 1963-2005 and portions thereof. For comparison, we also report estimates obtained using excess-returns on the 10 momentum portfolios as the test asset payoffs. The weighting matrix choice yields

CUE-GMM, which has nice statistical properties that allow for more efficient estimates. The use of excess-returns as test asset payoffs implies exact pricing of the risk free asset. By construction, excess-returns have a zero price. Therefore, in (15), we set $p_t = 0$.

Table 3 reports two sets of results. The first comprises point estimates of the b_1 and b_2 parameters of the QMFM and the p -values of the corresponding t -statistics constructed using standard errors based on our estimate of the spectral density matrix. The second set of results include the stock market covariance and coskewness annualized risk-premia $\lambda_{vw,1}$ and $\lambda_{vw,2}$, respectively, implied by the estimated $b_{vw,1}$ and $b_{vw,2}$ according to the beta-pricing representation, described by (A3), (A4) and (A5) in Appendix A, of the QMFM. The Table also reports p -values of the bootstrapped risk-premia distribution. The latter is generated using the so called “estimation-based” bootstrap introduced by Freedman and Peters (1984) and Peters and Freedman (1984)⁶. A comparison of Panel A and B of Table 3 shows that the role of coskewness risk in explaining pricing phenomena is emphasized when the 30 industry-sorted portfolios are used in place of the 17 industry-sorted ones. As shown by Figure 1, the frontier for the 30 industry-sorted portfolios lays noticeably above the frontier for the 17-industry sorted portfolios. This suggests that a positive price of coskewness risk helps generate the additional SR attainable by a finer industry-level diversification.

⁶ We first estimated time-series regressions of the test assets excess-returns on the factors (the stock market return proxy $R_{vw,t+1}$ and its square) and stored the residuals. We then re-sampled 5,000 times, with replacement, blocks of 5 consecutive realizations from the stored residuals time-series, i.e. we employed ‘block re-sampling’ to capture any residual serial correlation not explained by the estimated time-series regression. Using the time-series of the re-sampled residuals and the point estimates of the time-series regression parameters, we generated 5,000 separate bootstrapped return series for each test asset, for which we then re-estimated the QMFM model and recorded the $\lambda_{vw,1}$ and $\lambda_{vw,2}$ coefficients, calculated using (A5). This generates a bootstrapped distribution of the $\lambda_{vw,1}$ and $\lambda_{vw,2}$ coefficients.

As shown in Panel C of Table 3, the quadratic term coefficient is also highly significant when the momentum portfolios are used as the test asset payoffs. This is consistent with the findings of Harvey and Siddique (2000) and also in agreement with the conclusions reached by Rachev et al. (2007), who argue that momentum strategies entail considerable tail risk. The coskewness risk premium, however, is not statistically significant. In the terminology used by Cochrane (2005), this implies that the quadratic factor helps “price” the momentum portfolios but it is not “priced” in their cross-section. Interestingly, this suggests that a non-zero price of coskewness risk might help explain the momentum effect uncovered by Jagadeesh and Titman (1993) rather than the other way round. Noticeably, however, in spite of the extremely large amount of volatility of the estimated SDF, the quadratic model is rejected in all sample periods at the 5 percent level and at the 10 percent level in the period 1963-1993. In un-tabulated results, we find a similarly disappointing empirical performance for 4-moment versions of the model and for specifications that include human capital.

Another, and perhaps most noteworthy, fact highlighted by Table 3 is that the pricing ability of coskewness risk has diminished in recent times, as implied by the considerably reduced significance of both the price of coskewness risk and the coskewness risk premium in the 1963-2005 sample period relatively to the benchmark sample period used in Harvey and Siddique’s (2000) study, i.e. 1963-1993⁷. The less important role played by coskewness risk after the early 90s, while somewhat puzzling, might be explained by a number of circumstances. For example, the discovery by

⁷ Our estimated market coskewness risk premium for this period, when using the 30 industry-sorted portfolios as test assets, is -2.90 per cent per annum. It is thus very close to the -3.60 per cent per annum estimated by Harvey and Siddique (2000) and it is larger, in absolute value, than the market covariance risk-premium, which is just 2.70 per cent per annum. By contrast, in the extended sample period 1993-

Harvey and Siddique (2000) of the strength of the coskewness ‘effect’ might have led investors to engage in strategies designed to exploit it. The rise of hedge funds springs to mind as a possible key ingredient for such development. The analysis conducted by Zakamouline and Koekebakker (2009) suggests that hedge funds managers pursue strategies that exhibit negative skewness (and excess-kurtosis), for example by shorting options or engaging in dynamic portfolio insurance, in a quest for higher Sharpe Ratios. This behaviour can, in principle, drive down both the price of coskewness risk and the maximal SR attainable from the investment opportunity set, producing the drop in the SDF frontier that we observe in Figure 1.

On balance, these results clarify that our earlier findings (those reported in Table 1 and 2) do not contradict those of the extant literature but rather all such findings are sample-specific, in the sense that the estimated price of coskewness risk and the associated risk premium depend on the test asset payoffs used in estimation as well as on the sample period. Overall, coskewness appears to play a more important role in the pricing of strategies with relatively high SRs, such as those spanned by the finer industry level diversification allowed by the 30 industry portfolios, and in explaining returns on hard-to-price strategies, such as those spanned by the 10 momentum portfolios. The investigation of the determinants of the changing role of coskewness in asset pricing, as well as the difficult task of exploring the relations between momentum and coskewness, are outside the scope of this paper. In order to focus on the main task at hand, i.e. the testing of the CAPM and its higher moment versions, we leave these intriguing tasks to future research.

2005, the coskewness premium is -0.48 per cent per annum, and thus the coskewness discount is a mere 0.48 percent, whereas the covariance premium is a much larger 6.68 per cent per annum.

7. Conditional estimates

Next, we turn to conditional versions of (17). Initially, in (15) and (18), we use the full set of conditioning variables. In Table 4, we report the unconstrained estimation results, obtained using gross returns on the 17 industry portfolios and the risk-free asset as the basis test asset payoffs. The reported SDF coefficients are the values of a_t , $b_{vw,j,t}$ and $b_{l,j,t}$, with $j = [1, 2, 3]$, evaluated at the sample average values of the conditioning variables z_t . The associated p -values are significance levels of Wald tests of the restriction that the coefficient under consideration is zero. Consistently with Dittmar's (2002) findings, the non-linear SDF specifications achieve considerable empirical success. As shown in the lower panel of Figure 2, the estimated CMFM expected returns line up reasonably closely with sample average returns. Both the QMFM and the CMFM, but not the LMF, pass the Hansen and Jagannathan (1997) distance test when the market portfolio proxy includes human capital. The results imply that coskewness and cokurtosis risk are significantly priced in the cross-section of stock portfolios managed on the basis of available information. This contrasts with the insignificance of the coskewness and cokurtosis risk coefficients in unconditional estimates, suggesting that the quadratic and cubic terms generate the additional SDF volatility that, as shown in Figure 1, is required to price the managed portfolios used in conditional estimates.

The unrestricted estimates do not satisfy, however, a number of requirements of Kimball's (1993) standard risk aversion. For example, as shown in the top panel of Figure 2, the estimated SDF is not always positive and, as shown in the lower right-hand corner of the same figure, it is not decreasing in the return on the stock market

proxy. It may be argued that the estimated SDF takes negative values only very seldom, essentially only around times of exceptional market turmoil, namely the bear market that followed the 1974 oil shock, the 1987 stock market crash, the so-called 1997 Asian crisis and the crisis that followed the default of the Russian Federation on its domestic debt in 1998, as well as the bust of the “dot com” bubble in March 2000. Admittedly, these may be viewed as exceptional one-off circumstances, akin to outliers. The estimated SDF, however, is also very volatile, especially in the case of non-linear models. For example, the annualized volatility of the estimated SDF is 273 percent for the QMFM and 299 percent for the CMFM. Given a 15.5 percent annualized volatility of the stock market portfolio, this would imply that RRA takes values as large as 17.56 and 19.23, respectively. These values are puzzling when compared with the upper bound on RRA suggested by Ross (2005), i.e. $RRA_V = 5$, and survey evidence, as summarized by Meyer and Meyer (2005), which shows that, even for the most risk-averse households cohorts, RRA does not exceed 6.4 when the wealth portfolio includes only financial assets and 3.0 when it includes also non-financial assets.

To disentangle the pricing implications of violations of the NA positivity requirement from those of excessive SDF volatility, we estimate the candidate SDFs imposing each one of the two restrictions in turn. In Table 5, we report estimates obtained under a positivity restriction on the SDF. All candidate SDFs fail the Hansen and Jagannathan (1997) distance test when estimated under this restriction. This is also the case of non-linear specifications with human capital included in the market portfolio. Next, we estimate the candidate SDFs without the positivity restriction but under various upper bounds on their volatility. In Figure 3, we summarize the results of this exercise for the CMFM with human capital. In the Figure, we plot the model Hansen and Jagannathan

(1997) distance against the annualized volatility upper bound used in estimation and the corresponding RRA upper bound. All models estimated under a volatility upper bound set to less than 188 percent per annum are rejected at the 5 percent level. That is, it takes a RRA upper bound as loose as about 12.04 for the 4M-CAPM with human capital to be empirically admissible. These results are consistent with those reported by Dittmar's (2002), who also reject the 4M-CAPM as well as more restricted versions of this model. Our findings, however, clarify that the model is rejected not because of attempts to enforce overly restrictive requirements on the shape of the estimated utility function but rather, and more conclusively, because of the binding implications of sensible 'no good-deal' restrictions. In unreported estimates, we find this to be the case also in the earlier sample period used by Dittmar (2002), i.e. in the period 1963-1997.

These results imply that, while the prices of coskewness and cokurtosis risk are statistically significant, 3 and 4-moment versions of the CAPM provide an inadequate account of asset pricing patterns. In interpreting these findings, however, a caveat is in order. The test asset payoffs second moment matrix used to compute the weighting matrix of the moment conditions is close to singular. This implies that the pricing error metric being minimized in estimation is the pricing error of a portfolio that contains extreme long and short positions⁸. This is a widespread problem in empirical asset pricing and occurs also with the second moment matrix estimated using data for the period 1963-1997, i.e. the sample period used by Dittmar (2002). To mitigate this problem, we re-estimate using one conditioning variable at a time, i.e. including in z_t only one of the conditioning variables $r_{m,t}$, dy_t , ys_t , tb_t , as well as the unit-series. This

⁸ We verified this by computing the weights assigned by this matrix to the asset payoffs in the definition of the pricing error metric. These weights can be calculated by taking the sum by column of the elements

way, only 36 test asset payoffs at a time are used in estimation⁹. As shown in Table 6, the empirical performance of the CMFM with human capital improves somewhat and it passes the Hansen and Jagannathan (1997) distance test at the five percent significant level when the conditioning information variables set includes the lag of either the dividend yield, a term spread variable or the 3-month T-Bill return. The model, however, fails the test at the 10 percent in all cases except when the conditioning variables are the unit series and the lagged term spread and no restriction is placed on SDF volatility. When the conditioning variable is represented by the unit series and the lagged market excess return, the CMFM is rejected at the 0.1 percent level. These findings are important in that they confirm that our rejection of the 4M-CAPM is not a spurious by-product of a near-singular inverse weighting matrix. Interestingly, the fact that the model fares worst when the conditioning variable is represented by the lagged market excess return might be related to the well-known difficulty, also noted earlier, of explaining abnormal returns of momentum strategies.

8. Final remarks and conclusions

In this paper, we acknowledge the importance of Dittmar's (2002) findings and of the critique put forth by Post, Levy and Van Vliet (2008), in that they highlight the danger of spurious estimates of higher-moment versions of the CAPM. We emphasize, however, that a decreasing SDF, albeit sufficient, is not a necessary condition for the CAPM. Moreover, even if one is willing to restrict focus on the joint hypothesis of the higher moment CAPM and concave utility, the specification of the market portfolio

of the upper triangular matrix obtained from a Cholesky decomposition of the weighting matrix, i.e. of the inverse of the second moment matrix.

⁹ Reassuringly, the resulting second moment matrix is much less close to singular.

proxy may seriously affect inferences on the empirical admissibility of such hypothesis, much in the same way in which, as pointed out by Roll (1977), the circumstance that the market portfolio is *de facto* un-observable affects all tests of the CAPM. Recognising this difficulty amounts to acknowledging the ramifications of Roll's (1977) critique for tests of non-linear versions of the CAPM. In estimating the model, we thus impose alternative restrictions on the shape of the candidate SDF, namely a positivity requirement and a volatility upper bound. These restrictions boil down to ruling out arbitrage opportunities and SRs that, at least to the marginal investor, would resemble obvious near arbitrage opportunities¹⁰. This way, we limit the danger of over-fitting the cross-section of asset returns without the need to resort to the overly-restrictive assumption of concave utility.

Our findings confirm that the quadratic and cubic market factors help explain observed stock returns. They play an important role in the pricing of certain payoffs, including strategies characterized by relatively high SRs, such as those spanned by a fine industry-level diversification, most notably until the late 90s, or by dynamic portfolios managed on the basis of available conditioning information, as well as momentum portfolios. They do so, however, by generating high levels of SDF volatility. To rationalize this evidence within a higher moment CAPM framework, we would need to postulate implausibly high levels of investors' risk aversion. We conclude, therefore, that the 3M and 4M-CAPM provide at best a partial explanation of the differences in average returns on stocks and stock strategies. This gives rise to a coskewness (and

¹⁰ Beside, since coskewness is an asset characteristic that explains a considerable portion of the cross-section of asset returns, such an approach is also consistent with a multi-factor, no-arbitrage perspective, along the lines of Ross' (1976) APT. From this perspective, it is similar to the approach followed by Cochrane and Saà-Requejo (2000) and Cochrane (2005) to extend, in incomplete markets, the pricing implications of the factor prices and of (1) to a non-redundant security.

cokurtosis) puzzle. The solution of the latter requires an explanation, different from the 3M and 4M-CAPM, for why the quadratic and cubic market factors are priced in the cross-section of stock returns.

One obvious possibility is that these factors proxy for other priced but omitted factors. Vanden (2006) suggests that powers of the market returns proxy for omitted option-related factors. This possibility, while intriguing, requires further scrutiny because Vanden's (2006) sample period is relatively short and, more importantly, it remains to be established whether the SDF estimated by this author satisfies a sensible volatility upper bound. Another fruitful extension of our study would be a check of whether the more flexible specification used by Smith (2007)¹¹ is empirically admissible under appropriate positivity and volatility restrictions on the estimated SDF. An alternative approach would be to allow for a wedge between volatility of the candidate SDF and IMRS volatility by letting sentiment and variables related to investors' errors generate, as suggested by Shefrin (2008), the extra SDF volatility required to price stocks. There is also the possibility that some of the managed portfolios used in Dittmar's (2002) study and in ours correspond to unfeasible strategies, i.e. strategies with unfeasibly high SRs. Luttmer (1996), for example, shows how even modest proportional transaction costs, short sales restrictions and margin requirements considerably lower the mean-variance SDF frontier.¹² We leave the investigation of these possible explanations of the coskewness puzzle for future research.

¹¹ As shown by Smith (2007), his specification nests Dittmar's (2002) specification and hence the specification used in this paper.

¹² While we took this problem partially into account by re-estimating using one conditioning variable at a time, we feel that additional and more direct checks on the implications of transaction costs and market microstructure frictions might be worthwhile.

Appendix A: Alternative representations

The price of excess returns is by definition equal to zero. Thus, denoting by $r_{i,t+1}$ the excess return on the i -th asset, (1) can be rewritten as follows:

$$0 = E_t(m_{t+1}r_{i,t+1}) \quad (\text{A1})$$

Letting $f'_{t+1} = [R_{m,t+1} \quad R^2_{m,t+1} \quad R^3_{m,t+1}]$, the model in (2) can be rewritten more compactly as $m_{t+1} = a_t + b'_t f_{t+1}$. Then, the implications of (2) and (A1) for the cross-section of expected excess returns can be equivalently represented as follows:

$$E_t(r_{i,t+1}) = -\frac{\text{Cov}_t(r_{i,t+1}, f_{t+1})b_t}{E_t(m_{t+1})} = -(1 + R_{f,t})\text{Cov}_t(r_{i,t+1}, f_{t+1})b_t \quad (\text{A2})$$

$$E_t(r_{i,t+1}) = \beta'_{i,t} \lambda_t \quad (\text{A3})$$

Where,

$$\beta_{i,t} = \text{Var}_t(f_{t+1})^{-1} \text{Cov}_t(f_{t+1}, r_{i,t+1}) \quad (\text{A4})$$

$$\lambda_t = -(1 + R_{f,t})\text{Var}_t(f_{t+1})b_t \quad (\text{A5})$$

Here, variances and covariances are denoted by familiar symbols, $\beta_{i,t}$ is a vector of coefficients from the regression of asset i on the factors and the elements of the λ_t vector are the factor risk-premia. The expressions in (A2) and (A3) are known as the covariance and the beta-pricing representations of the implications of (2) and (A1).

Figure 1
SDF Mean-Variance frontier

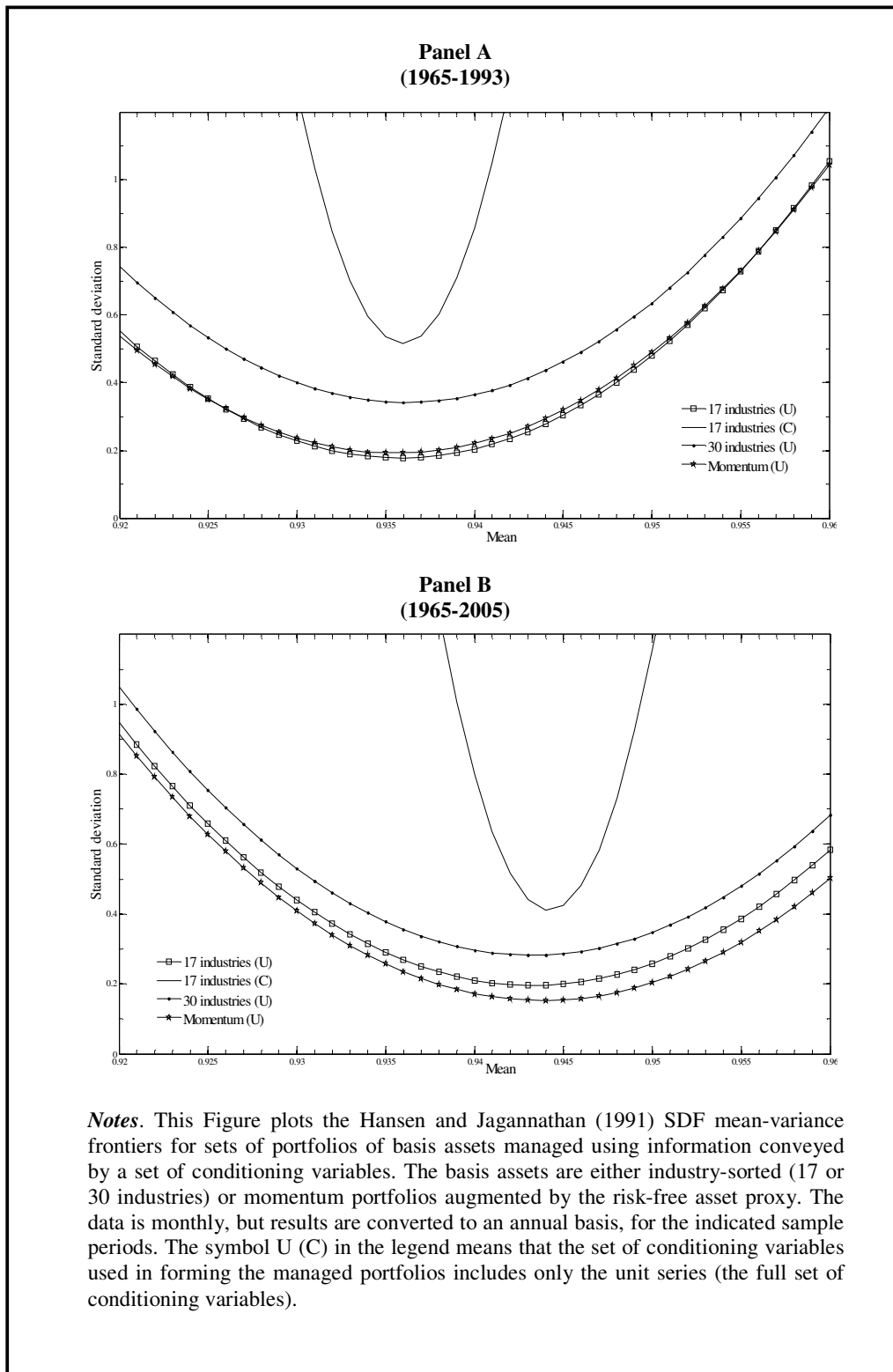


Table 1
GMM estimates of unconditional models
(1965-2005)

<i>Model</i>	<i>a</i>	<i>b_{yw,1}</i>	<i>b_{yw,2}</i>	<i>b_{yw,3}</i>	<i>b_{l,1}</i>	<i>b_{l,2}</i>	<i>b_{l,3}</i>	<i>HJ_T</i> (<i>p.-value</i>)	<i>σ(m)</i>	<i>RRA_v</i>
Panel A (Unconstrained estimates)										
<i>Market portfolio without human capital</i>										
LMFM	1.02 (0.000)	-2.55 (0.006)						0.18 (0.522)	0.40	2.55
QMFM	0.99 (0.000)	-2.57 (0.018)	1.25 (0.978)					0.17 (0.511)	0.40	2.57
CMFM	1.04 (0.000)	4.48 (0.138)	-34.48 (0.241)	-743.79 (0.039)				0.16 (0.677)	1.28	8.29
<i>Market portfolio with human capital</i>										
LMFM	0.985 (0.000)	-2.32 (0.040)			47.82 (0.321)			0.17 (0.567)	0.76	4.86
QMFM	1.01 (0.000)	-2.22 (0.096)	22.66 (0.687)		-3536.1 (0.736)	1783.12 (0.732)		0.17 (0.525)	0.88	5.67
CMFM	0.87 (0.001)	2.36 (0.303)	-14.60 (0.395)	-486.23 (0.154)	32.16 (0.262)	360.07 (0.373)	-516.15* (0.033)	0.12 (0.798)	1.86	11.99
Panel B (Estimates under NA)										
<i>Market portfolio without human capital</i>										
LMFM	1.02 (0.000)	-2.55 (0.006)						0.18 (0.522)	0.40	2.55
QMFM	0.99 (0.000)	-2.57 (0.018)	1.25 (0.978)					0.17 (0.511)	0.40	2.57
CMFM	1.03 (0.000)	3.68 (0.186)	-34.48 (0.241)	-743.80 (0.039)				0.17 (0.454)	1.12	7.22
<i>Market portfolio with human capital</i>										
LMFM	0.99 (0.000)	-3.21 (0.000)			-53.53 (0.000)			0.19 (0.292)	0.86	5.53
QMFM	1.26 (0.000)	-2.23 (0.017)	3.64 (0.470)		-46.32 (0.110)	-334.81 (0.361)		0.17 (0.420)	1.18	7.62
CMFM	0.86 (0.001)	1.77 (0.349)	-14.59 (0.395)	-486.23 (0.154)	32.17 (0.262)	360.07 (0.373)	-516.15* (0.033)	0.15 (0.428)	1.15	7.43

Notes. This Table reports unrestricted GMM estimates of the unconditional LMFM, QMFM and CMFM, obtained using Hansen and Jagannathan's (1997) second moment weighting matrix. For each included factor, we report the corresponding SDF coefficient point estimate and its p-value in brackets. We also report the Hansen and Jagannathan (1997) distance and its p-value in brackets. The second last column reports the annualized volatility of the stochastic discount factor in percentage. All the variables are defined as in the text. The test asset payoffs are gross returns on the 17 industry-sorted portfolios augmented by the gross return on the risk-free asset. The sample period is 1965-2005.

* ×100

Table 2
GMM estimates of unconditional models under NA, Local RA, NIARA and SDF
volatility restrictions
(1965-2005)

<i>Model</i>	<i>a</i>	<i>b_{vw,1}</i>	<i>b_{vw,2}</i>	<i>b_{vw,3}</i>	<i>b_{l,1}</i>	<i>b_{l,2}</i>	<i>b_{l,3}</i>	<i>HJ_T</i> (<i>p.-value</i>)	<i>σ(m)</i>	<i>RRA_v</i>
<i>σ(m) ≤ 0.78</i>										
<i>Market portfolio without human capital</i>										
LMFM	1.02 (0.000)	-2.55 (0.006)						0.18 (0.519)	0.40	2.55
QMFM	0.99 (0.000)	-2.57 (0.018)	1.25 (0.978)					0.17 (0.511)	0.40	2.57
CMFM	0.93 (0.000)	-2.61 (0.012)	45.64 (0.087)	-0.08 (0.974)				0.18 (0.376)	0.78	5.00
<i>Market portfolio with human capital</i>										
LMFM	1.01 (0.000)	-2.44 (0.010)			-0.01 (0.975)			0.17 (0.432)	0.37	2.39
QMFM	1.07 (0.000)	-0.71 (0.093)	1.39 (0.925)		-53.75 (0.000)	4795.55 (0.000)		0.18 (0.160)	0.78	5.00
CMFM	0.91 (0.000)	-0.37 (0.218)	1.91 (0.878)	-0.01 (0.925)	-20.47 (0.007)	4739.12 (0.000)	-73215.94 (0.893)	0.18 (0.140)	0.78	5.00

Notes. This Table reports GMM estimates of the unconditional LMFM, QMFM and CMFM for the period 1965-2005, obtained using Hansen and Jagannathan's (1997) second moment weighting matrix, under positivity and volatility restrictions on the estimated SDF. For each included factor, we report the corresponding SDF coefficient point estimate and its p-value in brackets. We also report the Hansen and Jagannathan (1997) distance and its p-value in brackets. The second last column reports the annualized volatility of the stochastic discount factor in percentage. All the variables are defined as in the text.

Table 3
QMFМ without human capital CUE-GMM estimates

<i>Sample Period</i>	<i>m > 0</i> (yes/no)	<i>R</i> ²	<i>b</i> _{vw,1} (<i>p</i> .-value) <i>λ</i> _{vw,1} (<i>p</i> .-value)	<i>b</i> _{vw,2} (<i>p</i> .-value) <i>λ</i> _{vw,2} (<i>p</i> .-value)	<i>TJ</i> _{<i>T</i>} (<i>p</i> .-value)	<i>σ</i> (<i>m</i>)	<i>RRA</i> _{<i>v</i>}
Panel A							
(17 industry portfolios)							
1963-2005	yes	0.01	-2.63 (0.010)	-8.28 (0.840)	0.18 (0.710)	0.43	2.77
			6.40 (0.000)	0.20 (0.500)			
1963-1993	yes	0.17	-2.59 (0.258)	87.48 (0.083)	0.14 (0.934)	1.35	8.85
			4.32 (0.000)	-1.96 (0.150)			
Panel B							
(30 industry portfolios)							
1963-2005	yes	0.05	-2.82 (0.027)	22.42 (0.572)	0.21 (0.761)	0.54	3.50
			6.68 (0.000)	-0.48 (0.270)			
1963-2000	yes	1.08	-3.94 (0.038)	78.49 (0.092)	0.22 (0.922)	1.26	8.29
			8.26 (0.000)	1.61 (0.250)			
1963-1997	yes	0.15	-4.18 (0.060)	116.95 (0.010)	0.21 (0.981)	1.73	11.68
			6.95 (0.000)	-2.33 (0.070)			
1963-1993	no	0.15	-2.25 (0.336)	127.96 (0.004)	0.21 (0.956)	1.94	12.70
			2.71 (0.000)	-2.90 (0.090)			
1963-1993	yes	0.12	-2.34 (0.278)	105.24 (0.000)	0.22 (0.930)	1.61	10.51
			3.39 (0.000)	-2.37 (0.050)			
Panel C							
(10 momentum portfolios)							
1963-2005	no	21.52	-2.67 (0.006)	-69.95 (0.145)	0.32 (0.000)	1.10	7.23
			6.66 (0.000)	1.48 (0.210)			
1963-2005	yes	0.03	-6.79 (0.000)	106.10 (0.000)	0.27 (0.002)	1.82	11.98
			15.14 (0.000)	-2.17 (0.150)			
1963-1993	no	0.22	-17.99 (0.003)	383.30 (0.004)	0.20 (0.057)	6.21	40.66
			34.45 (0.050)	-8.47 (0.300)			
1963-1993	yes	0.10	-8.02 (0.000)	108.55 (0.000)	0.35 (0.000)	1.96	12.87
			16.60 (0.050)	-2.34 (0.220)			

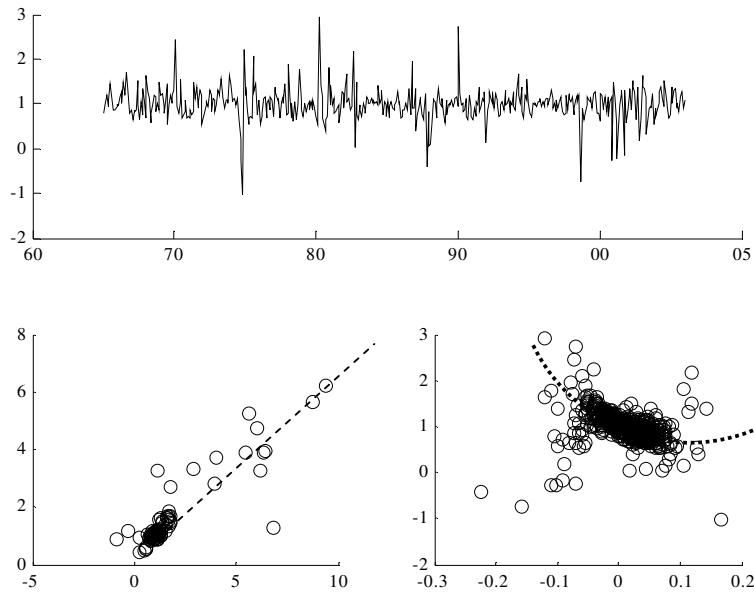
Notes. This Table reports CUE-GMM estimates of the unconditional QMFМ for various sample periods, specified in the first column. The second column indicates whether the estimated SDF satisfies the positivity requirement. The third column reports the coefficient of determination R^2 , i.e. the squared correlation coefficient between sample average excess returns and their model estimate. Next, we report the SDF coefficient point estimates with, in brackets, the p-values of their t-statistics computed using HAC standard errors. We also report the corresponding implied annualized percentage risk-premia and, in brackets, the p-values of their bootstrapped distribution. In the last 3 columns, we report Hansen's (1982) TJ_T statistic (and the corresponding p-value, in brackets), the estimated SDF volatility and the associated RRA bound. The test asset payoffs are monthly excess returns on the 17 and 30 industry-sorted and momentum portfolios.

Table 4
Unconstrained GMM estimates of conditional models
(1965-2005)

<i>Model</i>	<i>a</i>	<i>b_{vw,1}</i>	<i>b_{vw,2}</i>	<i>b_{vw,3}</i>	<i>b_{l,1}</i>	<i>b_{l,2}</i>	<i>b_{l,3}</i>	<i>HJ_T</i> (<i>p</i> - <i>value</i>)	<i>σ(m)</i>	<i>RRA_v</i>
<i>Market portfolio without human capital</i>										
LMFM	1.05 (0.000)	-3.67 (0.001)						0.53 (0.000)	0.73	4.69
QMFM	1.06 (0.000)	-5.80 (0.000)	24.25 (0.002)					0.50 (0.011)	1.24	7.96
CMFM	1.07 (0.000)	-2.03 (0.024)	5.73 (0.128)	-613.23 (0.605)				0.49 (0.012)	1.39	8.94
<i>Market portfolio with human capital</i>										
LMFM	1.04 (0.000)	-3.09 (0.005)			-1.96 (0.009)			0.50 (0.012)	1.60	10.27
QMFM	1.01 (0.000)	-4.14 (0.001)	38.46 (0.002)		-20.70 (0.039)	1,098.81 (0.041)		0.44 (0.228)	2.73	17.56
CMFM	0.96 (0.000)	-0.24 (0.030)	13.78 (0.544)	-548.68 (0.335)	-12.62 (0.106)	2,157.97 (0.249)	-75,667.48 (0.483)	0.42 (0.170)	2.99	19.23

Notes. This Table reports unrestricted GMM estimates of the LMFM, QMFM and CMFM with conditioning variables for the period 1965-2005, obtained using Hansen and Jagannathan's (1997) second moment weighting matrix. For each included factor, we report the corresponding SDF coefficient point estimate and its p-value in brackets. We also report the Hansen and Jagannathan (1997) distance and its p-value in brackets. The second last column reports the annualized volatility of the stochastic discount factor in percentage. All the variables are defined as in the text.

Figure 2
SDF
(1965-2005)



Notes. This Figure reports, in the top panel, the SDF time-series implied by estimates of the unconstrained CMFM and, in the bottom panel, the plot of the estimated expected return on the 17 industry sorted portfolios against their sample average return and the plot of the estimated SDF against the realizations of the market return sorted from lowest to highest. The data is monthly for the period 1965-2005.

Table 5
GMM estimates of conditional models under NA
(1965-2005)

<i>Model</i>	<i>a</i>	<i>b_{vw,1}</i>	<i>b_{vw,2}</i>	<i>b_{vw,3}</i>	<i>b_{l,1}</i>	<i>b_{l,2}</i>	<i>b_{l,3}</i>	<i>HJ_T</i> (<i>p</i> - <i>value</i>)	<i>σ(m)</i>	<i>RRA_v</i>
<i>Market portfolio without human capital</i>										
LMFM	1.05 (0.000)	-3.67 (0.001)						0.53 (0.000)	0.72	4.66
QMFM	1.06 (0.000)	-5.78 (0.000)	24.25 (0.005)					0.50 (0.007)	1.15	7.40
CMFM	1.06 (0.000)	-2.05 (0.064)	5.63 (0.178)	-613.24 (0.773)				0.50 (0.006)	1.21	7.81
<i>Market portfolio with human capital</i>										
LMFM	1.03 (0.000)	-3.26 (0.001)			0.65 (0.100)			0.52 (0.002)	1.17	7.54
QMFM	1.00 (0.000)	-4.86 (0.001)	38.00 (0.003)		-18.32 (0.199)	1,098.82 (0.521)		0.47 (0.024)	1.87	12.03
CMFM	0.92 (0.000)	-0.25 (0.049)	13.78 (0.804)	-548.68 (0.781)	-12.62 (0.873)	2,157.97 (0.596)	-75,667.48 (0.999)	0.46 (0.015)	2.30	14.81

Notes. This Table reports GMM estimates of the LMFM, QMFM and CMFM for the period 1965-2005, obtained using Hansen and Jagannathan's (1997) second moment weighting matrix under a positivity restriction on the estimated SDF. For each included factor, we report the corresponding SDF coefficient point estimate and its p-value in brackets. We also report the Hansen and Jagannathan (1997) distance and its p-value in brackets. The second last column reports the annualized volatility of the stochastic discount factor in percentage. All the variables are defined as in the text. The conditioning variables are $r_{m,t}$, dy_t , ys_t , tb_t .

Figure 3
Pricing errors vs. SDF volatility/RRA bound
Conditional CMFM with human capital
(1965-2005)

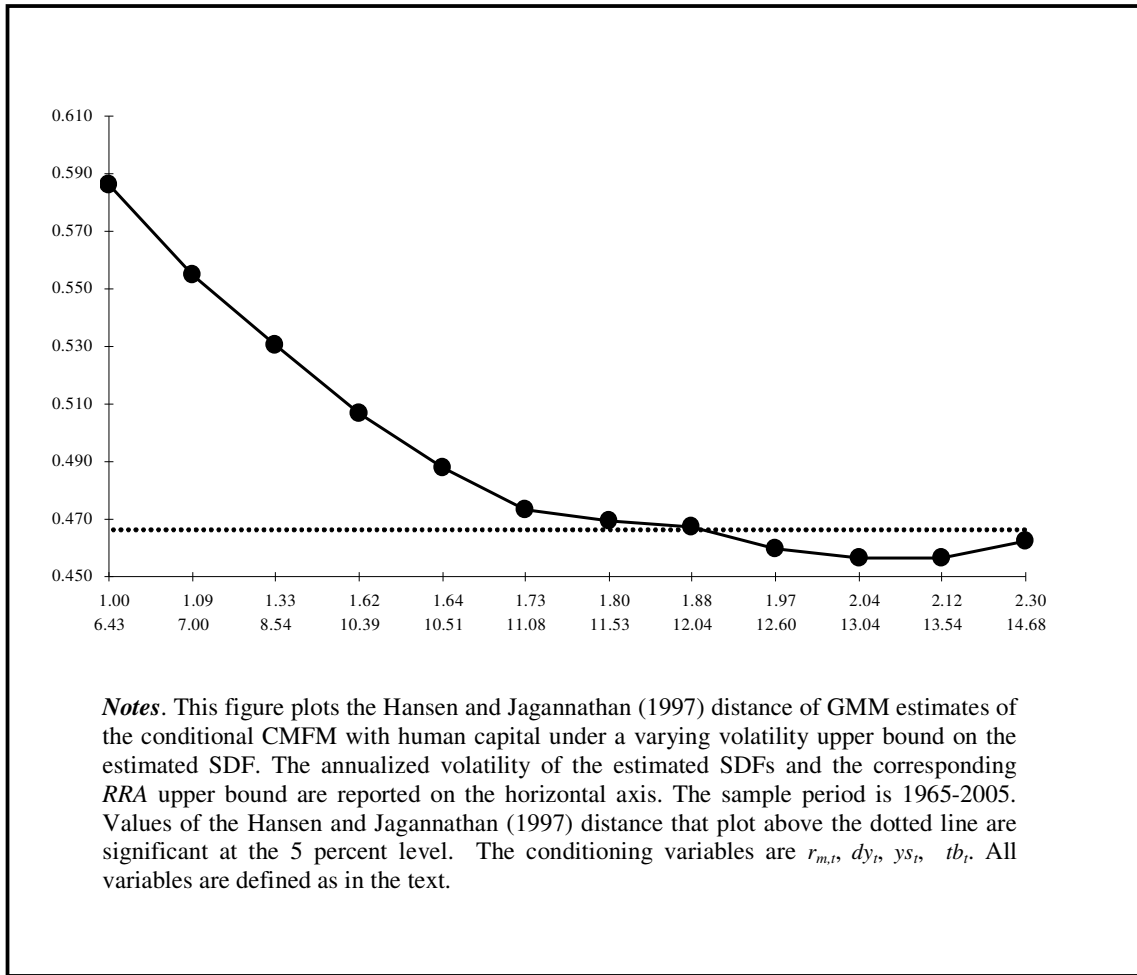


Table 6
GMM estimates of conditional CMFM under NA
(1965-2005)

z_t	a	$b_{vw,1}$	$b_{vw,2}$	$b_{vw,3}$	$b_{l,1}$	$b_{l,2}$	$b_{l,3}$	HJ_T (p.-value)	$\sigma(m)$	RRR_v
$r_{m,t}$	1.00 (0.000)	-2.67 (0.009)	0.00 (0.955)	-5.10 (0.759)	-5.58 (0.882)	1458.78 (0.504)	-45873.47 (0.842)	0.34 (0.001)	0.46	2.97
dy_t	1.21 (0.000)	-2.62 (0.020)	6.32 (0.758)	-25.35 (0.772)	-39.24 (0.040)	1313.89 (0.050)	-73356.49 (0.094)	0.29 (0.061)	0.74	4.77
ys_t	0.84 (0.000)	-2.23 (0.028)	0.13 (0.954)	-2.00 (0.907)	-10.43 (0.751)	4550.70 (0.011)	-5.09 (0.995)	0.27 (0.192)	0.94	6.07
ys_t	0.91 (0.000)	-0.58 (0.178)	39.34 (0.114)	-40.75 (0.228)	-1.85 (0.495)	1006.42 (0.049)	-77507.20 (0.045)	0.32 (0.005)	0.78	5.00
tb_t	0.86 (0.000)	-2.19 (0.032)	33.82 (0.283)	-2.88 (0.854)	-8.38 (0.686)	2739.05 (0.091)	-10212.62 (0.759)	0.30 (0.069)	0.78	5.00

Notes. This Table reports GMM estimates of the CMFM for the period 1965-2005, obtained using Hansen and Jagannathan's (1997) second moment weighting matrix under a positivity restriction on the estimated SDF. The first column reports the conditioning variable used in estimation. For each included factor, we report the corresponding SDF coefficient point estimate and its p-value in brackets. We also report the Hansen and Jagannathan (1997) distance and its p-value in brackets. The second last column reports the annualized volatility of the stochastic discount factor in percentage. All the variables are defined as in the text.

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