The Signature of Sentiment in Conditional Multi-Factor Model Estimates

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1. Introduction

Habit persistence, as in Campbell and Cochrane's (1999) model, and recursive non-(time)separable preferences, as those employed by Epstein and Zin (1989), go some way towards explaining the equity premium puzzle but rational asset pricing theory, based on the expected utility optimizing behavior of a representative investor that sets asset prices, has not yet fully succeeded in the task of explaining the cross-section of stock returns. For example, it is difficult to reproduce the explanatory power of a number of stock characteristics[1] using empirical specifications of available rational asset pricing models, most notably the CAPM. Well known examples of such characteristics are firms' size and their book-to-market ratio, as in Fama and French (1992, 1993) studies, Jagadeesh and Titman (1993) momentum and, more recently, idiosyncratic volatility, as in Ang, Hodrick, Xing and Zhang (2006).

Conditional specifications, henceforth (C)CAPM, that allow for time-variation in the parameters of the representative investors inter-temporal marginal rate of substitution (henceforth IMRS), and 3 and 4-moment versions, henceforth 3M-CAPM and 4M-CAPM, are amongst the most empirically successful extensions of the CAPM. For example, Lettau and Ludvigson (2001) demonstrate that a multifactor model inspired by the (C)CAPM performs much better than the unconditional CAPM and almost as well as the Fama and French (1992, 1993, 1995) 3-factor model, while

[1] Other examples of relatively successful characteristics are the momentum effect documented by Jegadeesh and Titman (1993) and the liquidity effect documented by Pastor and Stambaugh (2003).

Harvey and Siddique (2000) argue that a conditional version of the 3M-CAPM captures a large portion of the cross-sectional variation in average stock returns[2].

Lewellen, Nagel and Shanken (2006) and, more indirectly, Lewellen and Nagel (2006), however, warn that the sign of the risk premia estimated by Lettau and Ludvigson (2001) is problematic from the perspective of the conditional consumption CAPM. More specifically, the risk premia point estimates reported by Lettau and Ludvigson (2001) imply that (conditional) relative risk-aversion takes negative values for certain sample realizations of the conditioning variable. Moreover, these estimates imply a stochastic discount factor (henceforth, SDF), or pricing kernel, that might take negative values. This implies the existence of arbitrage opportunities, i.e. non-negative payoffs with a negative price. It is also inconsistent with the assumption that the representative investor's preferences display non-satiation, as in the (C)CAPM the SDF and the representative investor's IMRS coincide. There is therefore a puzzling contrast, in the specification of the (C)CAPM estimated by Lettau and Ludvigson (2001), between the high cross-sectional explanatory power of the factors implied by the model and the inconsistence between the parameter estimates and fundamental assumptions underlying the model itself. We might label this problem as the "(C)CAPM puzzle".

On a related note, Dittmar (2002) and Post, Levy and Van Vliet (2003) point out that

[2] Adesi, Gagliardini and Urga (2004) and Potì (2005), using a quadratic market model consistent with the 3M-CAPM, add to the evidence that models that allow for both covariance and coskewness premium fit the cross-section of stock returns well.

covariance and coskewness risk prices estimated in empirical tests of the 3 and 4 moment CAPM imply a non-concave utility function, to an extent that might be inconsistent with the models being tested. More worryingly, these authors show that the empirical fit of these models is greatly reduced when the shape of the representative investor's utility function is restricted to display non satiation, risk aversion and non increasing absolute risk aversion (henceforth, NS, RA and NIARA, respectively). Yet, the evidence on the cross-sectional explanatory power of coskewness is compelling. This evidence, coupled with the critique put forth by Post, Levy and van Vliet (2004) and Potì (2005), gives rise to a "coskewness puzzle" in the complete market-representative investor setting of the 3M-CAPM.

There is the concrete possibility that an omitted factor problem might be behind both puzzles. In this case, OLS estimates of the model parameters, i.e. factor risk premia and risk prices, would be inconsistent. In this paper, we explicitly explore this possibility and, in particular, we investigate whether the omitted factors are related to systematic investor error. We thus propose a multi-factor specification that augments 2 and 3 moment versions of the (C)CAPM with a sentiment factor. This specification is based on Shefrin's (2005) central result, namely that the pricing kernel can be decomposed into two terms, one being sentiment and the other being an expression that depends only on economic fundamentals. The former term can be seen as the behavioural component of the kernel, while the second can be seen as its rational component. We check whether adding the sentiment factor, and thus allowing for a behavioural influence on asset pricing, allows to retain or improve the explanatory power of the (C)CAPM and 3M-CAPM while admitting risk price point estimates consistent with investors risk aversion (RA) and non increasing absolute risk aversion (NIARA), and thus with the underlying economic theory. Baker and Wurgler (2006) carefully constructed a proxy for investors' sentiment and demonstrated that exposure to this variable can explain a significant portion of the cross-section of stock returns. We thus employ this variable to proxy for sentiment in our specification.

The remaining of this paper is organized as follows. In the next Section, we introduce Shefrin's (2005) SDF decomposition into a rational and sentiment related component and we map this decomposition into a linear factor model. In Section 3, we outline our estimation strategy. In Section 4, we present our dataset. In Section 5, we report our empirical results. Section 6 concludes.

2. Sentiment and the Pricing Kernel

Let the kernel m_{t+1} price the traded payoffs x_{t+1}:

$$p_t = E_t\left(m_{t+1}x_{t+1}\right) \tag{1}$$

Here, p_t denotes payoff prices and the expectation is taken conditional on the available information set. Assuming a representative investor's utility function that satisfies non satiation (NS), risk aversion (RA) and non increasing absolute RA (NIARA) and following Shefrin (2005), we may model the stochastic process for the pricing kernel as follows:

$$m(x_i) = \delta_R \Lambda(x_i) \frac{U'(C(x_i))}{U'(C_0)} \tag{2}$$

Here, δ_R is the representative investor's subjective rate of time preference, $\Lambda(x_i) \equiv \frac{P_R(x_i)}{\Pi(x_i)}$, $P_R(x_i)$ represents the probability weights in state x_i of the representative trader, $\Pi(x_i)$ is the objective probability of occurrence of state x_i, C_0 represents current aggregate consumption and $C(x_i)$ represents aggregate consumption in state x_i. Letting for simplicity $\delta_R \cong 1$ and exploiting the fact [3] that $\Lambda(x_i) \cong 1 + \ln \Lambda(x_i)$, the kernel process can be approximated as $m(x_i) \cong [1 + \ln \Lambda(x_i)] \frac{U'(C(x_i))}{U'(C_0)}$ or

$$m(x_i) \cong \frac{U'(C(x_i))}{U'(C_0)} + \frac{U'(C(x_i))}{U'(C_0)} \ln \Lambda(x_i) = m_c + m_s \tag{3}$$

In (3), $m_c \equiv \frac{U'(C(x_i))}{U'(C_0)}$ is aggregate marginal utility growth, while $m_s \equiv \frac{U'(C(x_i))}{U'(C_0)} \ln \Lambda(x_i)$ is the product of marginal utility growth and the log of the

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[3] This approximation implicitly assumes that $\Lambda(x_i)$ is never much different from one.

ratio between the representative investor's probabilities and the correct probabilities. The above equation thus states that the log-kernel is the sum of two distinct processes, one based on the kernel of a rational expected utility optimizing representative investor and the other based on sentiment. The sentiment component is zero only when the likelihood ratio $\Lambda(x_i)$ equals one and thus only when prices are set in such a way that the representative investor holds objectively correct belief.

We may approximate the marginal utility growth of the representative investor with preferences defined over aggregate consumption using a Taylor's expansion,

$$\frac{U'(C(x_i))}{U'(C_0)} = 1 + \frac{1}{2}\frac{U''(C_0)}{U'(C_0)} C_0 R_c(x_i) + \frac{1}{6}\frac{U'''(C_0)}{U'(C_0)} C_0^2 R_c(x_i)^2 + \ldots \qquad (4)$$

Here, $R_c(x_i) \equiv \frac{\Delta C(x_i)}{C_0}$ is aggregate consumption growth. Following Dittmar (2002), we might allow for a multi-period setting with predictability due to a possibly time-varying investment opportunity set, and we might thus generalize (4) as follows:

$$m_{c,t+1} = 1 + h_{1t}\frac{U''(C_t)}{U'(C_t)} R_{ct+1} + h_{2t}\frac{U'''(C_t)}{U'(C_t)} R_{ct+1}^2 + h_3\frac{U''''(C_t)}{U'(C_t)} R_{ct+1}^3 + \ldots \qquad (5)$$

Here, the h_i terms, i = 1, 2 3…, are non-negative expansion parameters. Dropping terms of order higher than the second, we might rewrite $m_{c,t+1}$ more compactly as follows:

$$m_{c,t+1} = 1 + b_{1t} R_{ct+1} + b_{2t} R_{ct+1}^2 \ldots \tag{6}$$

Since $b_{1t} \equiv h_{1t} \dfrac{U^{''}(C_t)}{U^{'}(C_t)}$ and $b_{2t} \equiv h_{2t} \dfrac{U^{'''}(C_t)}{U^{'}(C_t)}$, RA and NIARA imply $b_{1t} < 0$ and $b_{2t} \geq 0 \;\; \forall$ *t*. With power utility with risk aversion parameter γ, we have that $b_{1t} = -\gamma$ while the coefficients of the higher order terms are zero. This is the constant relative risk aversion (CRRA) case considered by Sheffrin (2005).

The component m_s of the kernel, based on (3), is the cross-product of m_c and the log-likelihood ratio $\ln \Lambda(x_i)$. Letting, for notational simplicity, $\ln \Lambda(x_i) = s(x_i)$, using (3) and (6), and dropping terms of third and higher orders, we can thus rewrite the kernel as follows:

$$m_{t+1} \cong 1 + b_{1t} R_{ct+1} + b_{2t} R_{ct+1}^2 + b_3 s_{t+1} R_{ct+1} \tag{7}$$

3. Estimation Strategy

Consider the general conditional factor model, where the SDF is a (conditionally) linear function of a set of factors f_{t+1},

$$m_{t+1} = a_t + b_t' f_{t+1} \tag{8}$$

The model in (7) is a specification of the general conditional factor model in (8), with $a_t = 1$ and factors f_{t+1} given by a constant, the growth rate of aggregate consumption, R_{ct+1}, the square of the latter, R^2_{ct+1} , and the cross-product between the sentiment proxy, s_{t+1}, and the consumption growth rate, $s_{t+1}R_{ct+1}$. Since the price of excess returns $r_{i,t+1}$ is by definition zero, (1) can be rewritten as $0 = E_t\left(m_{t+1}r_{i,t+1}\right)$. The unconditional implications of the latter are

$$0 = E\left(m_{t+1}r_{i,t+1}\right) \tag{9}$$

As in Dittmar (2002), we model variation in b_t as a linear function of the first lag of a vector of conditioning variables z_t (which typically include a constant), i.e.

$$b_t = b_i' z_t \qquad i = 1,\ldots k \tag{10}$$

The unconditional factor model implied by (8), (9) and (10) can then be written as follows,

$$m_t = a + b\ 'F_{t+1} \tag{11}$$

Here, $\mathrm{a} = 1 - b' E(f_{t+1} \otimes z_t)$ and b is a vector that stacks all the b_i vectors in a column. The specification in (11) is an unconditional model, i.e. a model with time-invariant parameters, in the new set of (unconditionally) de-meaned factors $F_{t+1} = (f_{t+1} \otimes z_t) - E(f_{t+1} \otimes z_t)$. When pricing excess returns, the means of excess returns do not identify the mean of the risk free rate[4], see for example Cochrane (2001). Thus, for simplicity, we will set the mean of the kernel in (11) equal to one,

$$E(m_{t+1}) = a + b'E(f_{t+1}) \cong 1 \tag{12}$$

Based on (11) and (12), we can rewrite the restrictions that (1) and (9) impose on the cross-section of expected returns as follows:

$$E\left(r_{i,t+1}\right) = \beta_i' \lambda \tag{13}$$

Here, $\beta_i = Var\left(F_{t+1}\right)^{-1} Cov\left(F_{t+1}, r_{i,t+1}\right)$ is a vector of factor loadings of the regression of asset i on the factors. The elements of λ are factor risk-premia (of the unconditional model). Following a widely used terminology, we will refer to (13) as the beta-pricing representation of the restrictions that (9) and (11) impose on the cross-section of expected returns. The risk-premia and the parameters of the unconditional SDF model are linked as follows:

$$\lambda \cong -Var\left(F_{t+1}\right)b \tag{14}$$

We estimate (13) using a robust 2-pass regression without intercept in the second pass. In the first pass of this procedure, we regress the test asset payoff excess returns on the factors of the unconditional model and in the second pass we regress average excess

4 This, however, is strictly true only as long as the risk free rate is not unrealistically high.

returns on the factor loadings estimated in the first pass. This yields factor risk premia λ estimates and we then retrieve the parameters of the SDF (of the unconditional model) using (14). It can be shown that this approach is equivalent to first-stage GMM. In a GMM setting, in fact, the parameters of the kernel that prices a vector of *n* test asset payoff x_{t+1} can be estimated by solving the following problem:

$$\min_{\{m\}} \quad g_T' W_{n(1+k)\times n(1+k)} g_T \tag{15}$$

$$g_T = E_T(g_t) \qquad g_t = [m_{t+1}(b)x_{t+1} - p_t] \otimes z_t \tag{16}$$

Here, $E_T(.)$ denotes a sample average, i.e. an arithmetic average over a sample of *T* observations, and z_t is a vector of *k* instruments that coincide with the conditioning variables in (10). Based on (9), the elements of the $n\times 1$ vector g_t can be interpreted as pricing errors and the moment conditions g_T as pricing errors sample averages. Under the usual ergodicity assumption, the latter are consistent estimates of the unconditional expectations of (9). The $n\times n$ matrix *W* is a weighting matrix for the moment conditions. The 2-pass OLS regression amounts to minimizing (15) using the identity matrix to weight the moment conditions, i.e. setting *W* = *I*. Efficient second-stage and iterated GMM estimates are obtained instead by setting *W* equal to Hansen's (1982) optimal weighting matrix. We also experiment with the latter approach but we do not report the results as these are qualitatively the same as those obtained using the 2-pass regression. Finally, with the parameters of the unconditional model in hand, we back out the parameters of the conditional model using (10).

4. Data

The sample period starts in the first quarter of 1966 (to avoid the impact of dividend taxation reform in the early part of the previous decade) and ends in the last quarter of 2005. The test asset payoffs are 25 size and book-to market sorted portfolio and 30 value-weighted sorted portfolios of the NYSE, AMEX, and NASDAQ stocks[5]. We use returns on the three-month Treasury Bill as the risk-free rate and aggregate consumption expenditure data constructed as in Lettau and Ludvigson (2001)[6]. We set the elements of s_t equal to the first difference of a proxy for investors' sentiment constructed as described by Baker and Wurgler (2006)[7], namely the updated version of their $\Delta SENT^{\perp}$ variable. More specifically, the latter is the first difference of a sentiment index[8] based on the first principal component of six (standardized) sentiment proxies over the period 1966-2005, where each of the proxies has first been orthogonalized with respect to a set of macroeconomic conditions. Finally, we use the consumption-wealth ratio estimates cay_t supplied by Lettau and Ludvigson (2001) as the conditioning variable that drives, in (10), the variation of the parameters of the representative investor's IMRS. Notice that the much debated "look-ahead" bias of this variable, if present, is actually a desirable feature in our context. This is because rational expectations themselves should indeed display look ahead bias, as rational

5 Data on this portfolios was downloaded from Kenneth French website.
6 We thank Martine Lettau and Sidney Ludvigson for making this data available on their web-sites.
7 We thank these authors for making this data available for download from the AFA-Journal of Finance web-site.
8 This index is given by Equation (3) in their paper.

investors know the return data generating process, see Muth (1961) for a seminal reference and, more recently, Sargent (1993).

5. Empirical Results

Table 1 and 2 report estimates of an empirical specification of the (C)CAPM and 3M(C)CAPM, with an without sentiment. In particular, Table 1 reports risk premia estimates and associated t-statistics. We construct t-statistics using standard errors adjusted, based on Shanken's (1992) correction, for sampling error that arises because the regressors β_t are estimated in a first-stage time-series regression. Following Lettau and Ludvigson (2001), we also construct conventional OLS t-statistics. This is motivated by the argument put forth by Jagannathan and Wang (1996), who show that OLS standard errors from a 2-pass procedure do not necessarily overstate the precision of the standard errors, even if conditional heteroskedasticity is present. Table 2 reports the estimates of the SDF parameters (of the unconditional model), their t-statistics, without and with Shanken's (1992) correction, and a first-stage GMM *TJ* test statistic of the null that the pricing errors are jointly zero. This statistic is constructed as a quadratic form of the pricing errors $g_T' W g_T$, weighted by the first-stage GMM weighting matrix, $W = Cov(g_T(\hat{b}))^{-1}$. The latter is defined as follows

$$TCov(g_T(\hat{b})) = \left[I_n - d(d'd)^{-1}d'\right]' Cov_T(g_t)\left[I_n - d(d'd)^{-1}d'\right] \qquad (17)$$

Here, I_n is the $n \times n$ identity matrix, $Cov_T(.)$ denotes a sample variance-covariance matrix and $d = \frac{\partial g_T(b)}{\partial b} = E_T(F_t r_t')$ is the vector of the first derivatives of the moment conditions in (16) with respect to the model parameters.

Concerning the (C)CAPM estimates, all the factor risk premia are statistically significant when t-statistics are computed using OLS standard errors. With Shanken's (1992) standard error adjustment, however, only the risk premium of $cay_t R_{ct+1}$ is statistically significant. This should not surprise as it simply means that there is considerable sampling error in the estimation of the factor loadings β_t. The coefficient of determination is, however, not very high. In particular, it is much lower than the coefficient of determination reported by Lettau and Ludvigson (2001). This is due to the fact that they include an intercept in the second pass regression, thus reducing pricing errors. More importantly, the estimates of b imply that both the SDF and b_{1t} display the wrong sign for prolonged portions of the sample period. As shown in Panel A of Figure 1, the estimated SDF often takes negative values, thus assigning negative prices to non-negative payoffs almost surely. This implies the existence of arbitrage opportunities and violation of the assumption that investors preferences display non-satiation. Moreover, as shown in Panel B of Figure 1, the estimated b_{1t} does not always take a negative value, thus implying a violation of the assumption that investors are risk averse. Interestingly, b_{1t} becomes positive roughly when anecdotal evidence suggests that the market assessment of perspective earnings growth might have been affected by over-optimism, i.e. in the second part of the 1990s, end it peaks on June 2000, roughly at the beginning of the stock market correction. The estimates of the 3M(C)CAPM suffer from the same problems, i.e. the SDF is not always positive and b_{1t} is not always negative (neither however plotted to save space), alongside a non-positive point estimate of b_2, in contrast with the assumption of non-increasing absolute risk aversion and thus with standard risk aversion.

The Sentiment-(C)CAPM and Sentiment-3M(C)CAPM are empirically more successful, and most of the increase in the explanatory power is due to the inclusion of sentiment. As shown in Table 1, the R^2 of Sentiment-(C)CAPM jumps to 58.9 (50.7 adjusted). This value is comparable to the coefficient of determination obtained by Lettau and Ludvigsn (2001) allowing for an intercept in the second pass regression. The R^2 of the 3-moment (C)CAPM with sentiment is not much larger (it is actually lower, when adjusted for degrees of freedom) than the R^2 of its 2-moment version but it is much larger than the 3-moment specification without sentiment.

The *TJ* test statistic reported in Table 2 is never statically significant, thus implying that pricing errors are jointly insignificant. Moreover, while not all risk premia are statistically significant at conventional levels (merely implying that the price of the factors themselves might be close to zero), both aggregate consumption and sentiment are priced in the cross-section of average returns, as demonstrated by the significance of the corresponding elements of the *b* vector reported in Table 2. More importantly, the point estimates of the elements of ***b*** imply a much better behaved behaviour of the rational component of the SDF compared to the specifications without sentiment. As shown in Panel A of Figure 2, while the SDF still often takes negative values, its rational component (the ticker line) is rarely negative. Perhaps more remarkably, b_{1t} is now always negative, both in the case of the 2-moment model, as shown in Panel B of Figure 2, and in the case of the 3-moment model (not reported to save space). This is important as it implies that, including sentiment as a factor, the sign of the price of systematic conditional covariance is consistent with the risk-aversion assumption for any realization of the conditioning variable. In Figure 3, we report the sentiment component of the SDF. This is the component that helps the SDF fit the cross-section of average excess returns while allowing for a representative investor's IMRS much more consistent with the tenets of rational optimizing behaviour. As it is evident from the Figure, this component of the SDF peaks at times of high market valuations, such as the so called 'dot-com' boom of the late 1990s, and its troughs coincide with the bottom of market corrections, when investors' judgment might have been clouded by pessimism, as in the aftermath of the Latin American debt crisis in 1982. At these times, it is necessary to allow for a substantial amount of systematic investors' information processing error, to be able to justify stock valuations and still retain the assumption that, by and large, investors are greedy, risk averse, expected utility maximizing individuals.

We also report estimates that add an extra sentiment factor, namely s_t alongside $s_t cay_{t\text{-}1}$, and, to facilitate comparison of our estimates with those reported by Lettau and Ludvigson (2001), we include $cay_{t\text{-}1}$ among the factors. The estimates of these augmented specifications are reported in Table 3 and 4. The only qualitatively important difference is that now the point estimate of the b_2 parameter of the 3M-(C)CAPM is positive, in accordance with NIARA. This suggests that there might be additional factors with which both $cay_{t\text{-}1}$ and the square of the consumption growth rate are correlated. We leave for future research the identification of the extra factor (factors) and, more importantly, of whether it is should enter the rational or the sentiment-related component of the SDF.

6. Conclusions

This paper shows that augmenting the (C)CAPM with sentiment, and thus allowing for systematic investor error in forming beliefs about the distribution of returns, allows to largely reconcile investors' optimizing behaviour with the cross-section of average returns. This implies that investors must either commit systematic errors, at least ex-post, in assessing the joint distribution of stock returns and aggregate consumption or they must behave in a way that, at the aggregate level, is inconsistent with expected utility maximization and with standard risk aversion assumptions, as formulated for example by Kimball (1993). Future research might ascertain whether these systematic (ex-post) errors might have been avoided making full use of available information, thus implying a violation of the Efficient Market Hypothesis, as formulated by Fama (1970, 1976).

Table 1
Second Pass Regressions (1966-2005) Estimates
25 Size and Book-to-Market Portfolios
Beta-Pricing Representation

Model	R_{ct+1}	cay_tR_{ct+1}	$s_{t+1}R_{ct+1}$	R_{ct+1}^2	R^2 (Adj. R^2)
(C)CAPM	0.48	0.02			26.1
	(3.21)	(5.97)			(22.8)
	(0.99)	(1.83)			
3M(C)CAPM	0.21	0.01		0.01	33.7
	(1.36)	(4.71)		(4.63)	(27.6)
	(0.45)	(1.53)		(1.57)	
Sentiment-(C)CAPM	0.84	0.01	-1.35		56.5
	(5.59)	(3.70)	(-6.22)		(52.6)
	(1.76)	(1.21)	(-1.96)		
Sentiment-3M(C)CAPM	0.67	0.01	-1.22	0.01	56.5
	(4.61)	(3.25)	(-5.74)	(5.00)	(50.3)
	(1.59)	(1.16)	(-1.99)	(1.78)	

Notes. This Table reports 2-step regression estimates of the beta-pricing representation of various factor models for the period 1966-2005. The second pass regressions are estimated without an intercept term. The top row indicates the factors included in each model. For each included factor, we report the risk premia point estimates in percentage and two sets of t-statistics in brackets. These statistics are computed using OLS standard errors that account for correlated errors across test portfolios while the second set of t-statistics also uses Shanken' (1992) correction for the fact that the beta coefficients are estimated. The third and second last columns report the percent coefficient of determination R^2, both unadjusted and adjusted for the degrees of freedom. All the variables are defined as in the text. The data frequency is quarterly.

Table 2
Second Pass Regressions (1966-2005) Estimates
25 Size and Book-to-Market Portfolios
Implied SDF Parameters

Model	R_{ct+1}	cay_tR_{ct+1}	$s_{t+1}R_{ct+1}$	R_{ct+1}^2	$TJ_{Shanken}$	$\sigma(m)$	$\sigma(m_c)$ $\sigma(m_s)$
(C)CAPM	-418.00	-37,904.00			17.00	629.8	629.8
	(0.000)	(0.000)			(0.763)		629.8
3M(C)CAPM	284.00	-30,117.00		-70,956.00	24.28	632.2	632.2
	(0.399)	(0.003)		(0.038)	(0.230)		632.2
Sentiment-(C)CAPM	-526.00	-13,219.00	305.00		15.08	617.5	446.8
	(0.000)	(0.309)	(0.007)		(0.772)		383.5
Sentiment-3M(C)CAPM	-209.00	-14,153.00	252.00	-30,205.00	24.25	433.84	239.25
	(0.987)	(0.319)	(0.056)	(0.450)	(0.232)		316.76

Notes. This Table reports the elements of the *b* vector, the negative of the risk prices, implied by 2-pass regression estimates (without intercept in the second pass regression) and, in brackets, associated p-values. These are computed using standard errors based on a weighting matrix equal to the inverse of the sample (first stage) estimate of the pricing errors variance-covariance matrix. The same weighting matrix is used in the computation of the average pricing errors *TJ* reported in the third last column. The last two columns report the annualized volatility of the estimated stochastic discount factor (in percentage) and its decomposition in a rational and sentiment-related component. The market Sharpe ratio is about 40 percent. All the variables are defined as in the text. The data frequency is quarterly.

Table 3
Augmentation by cay_t and s_t
Beta-Pricing Representation

Model	cay_t	R_{ct+1}	cay_tR_{ct+1}	s_{t+1}	$s_{t+1}R_{ct+1}$	R_{ct+1}^2	R^2 (Adj. R^2)
1	2.34	0.41	0.02				22.8
	(4.86)	(2.60)	(6.41)				(15.8)
	(1.49)	(0.79)	(1.93)				
2	0.86	0.64	0.01			0.003	34.3
	(2.23)	(3.87)	(4.22)			(2.13)	(24.9)
	(0.74)	(1.27)	(1.39)			(0.71)	
3	1.16	0.71	0.01		-1.46		54.7
	(3.18)	(4.54)	(4.20)		(-6.43)		(48.2)
	(1.34)	(1.71)	(1.52)		(-3.10)		
4	1.09	0.65	0.01		-1.39	0.01	55.16
	(3.32)	(4.44)	(3.59)		(-7.91)	(4.20)	(46.2)
	(1.02)	(1.47)	(1.44)		(-2.11)	(1.43)	
5	0.84	0.60	0.01	-80.61	-1.12		58.9
	(2.59)	(3.93)	(4.36)	(-3.36)	(-6.45)		(50.7)
	(0.97)	(1.43)	(1.62)	(-1.25)	(-2.38)		
6	0.86	0.64	0.01	-74.86	-1.13	0.003	59.2
	(2.61)	(4.33)	(4.41)	(-3.19)	(-6.42)	(2.13)	(48.5)
	(0.92)	(1.49)	(1.52)	(-1.12)	(-2.24)	(0.73)	

Notes. This Table reports 2-step regression estimates of the beta-pricing representation of various factor models for the period 1966-2005. The second pass regressions are estimated without an intercept term. The top row indicates the factors included in each model. For each included factor, we report the risk premia point estimates in percentage and two sets of t-statistics in brackets. These statistics are computed using OLS standard errors that account for correlated errors across test portfolios while the second set of t-statistics also uses Shanken' (1992) correction for the fact that the beta coefficients are estimated. The third and second last columns report the percent coefficient of determination R^2, both unadjusted and adjusted for the degrees of freedom. All the variables are defined as in the text. The data frequency is quarterly.

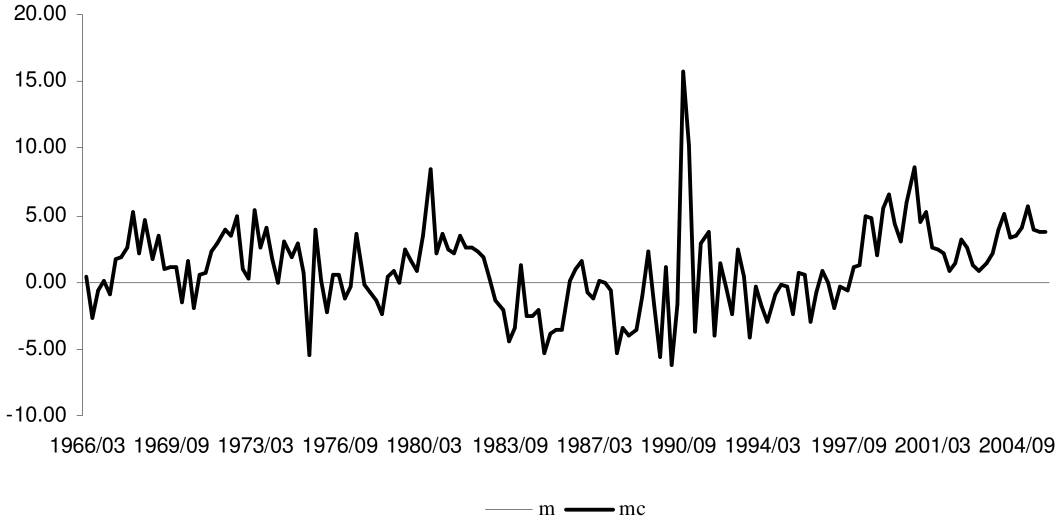
Table 4
Augmentation by cay_t and s_t
Implied SDF Parameters

Model	cay_t	R_{ct+1}	cay_tR_{ct+1}	s_{t+1}	$s_{t+1}R_{ct+1}$	R_{ct+1}^2	$TJ_{Shanken}$	σ(m)	σ(m_c) σ(m_s)
1	-21.67	-391.61	-36,754.60				24.14	636.6	636.6
	(0.655)	(0.000)	(0.000)				(0.340)		636.6
2	-31.00	-510.00	-10,945.00			14,812.00	25.34	364.2	364.2
	(0.536)	(0.132)	(0.284)			(0.672)	(0.189)		364.2
3	-55.40	-468.80	-7,893.20		334.00		16.24	616.8	409.2
	(0.270)	(0.000)	(0.568)		(0.004)		(0.756)		419.9
4	-48.00	-342.00	-8,977.00		308.00	-12,772.00	16.23	544.2	332.4
	(0.393)	(0.448)	(0.531)		(0.037)	(0.775)	(0.702)		387.2
5	-27.96	-381.17	-11,457.46	-1.22	387.55		20.46	524.9	350.3
	(0.633)	(0.002)	(0.426)	(0.368)	(0.003)		(0.429)		340.3
6	-31.00	-510.00	-10,945.00	-1.00	429.00	14,812.00	12.56	556.6	298.91
	(0.600)	(0.297)	(0.451)	(0.618)	(0.033)	(0.785)	(0.895)		410.82

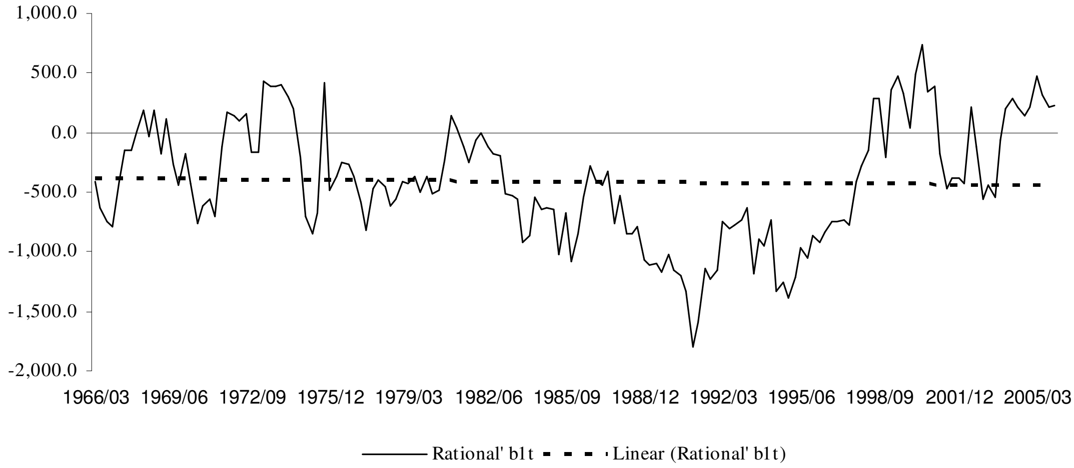
Notes. This Table reports the elements of the *b* vector, the negative of the risk prices, implied by 2-pass regression estimates (without intercept in the second pass regression) and, in brackets, associated p-values. These are computed using standard errors based on a weighting matrix equal to the inverse of the sample (first stage) estimate of the pricing errors variance-covariance matrix. The same weighting matrix is used in the computation of the average pricing errors *TJ* reported in the third last column. The last two columns report the annualized volatility of the estimated stochastic discount factor (in percentage) and its decomposition in a rational and sentiment-related component. The market Sharpe ratio is about 40 percent. All the variables are defined as in the text. The data frequency is quarterly.

Figure 1
(C)CAPM

Panel A
(Estimated SDF)



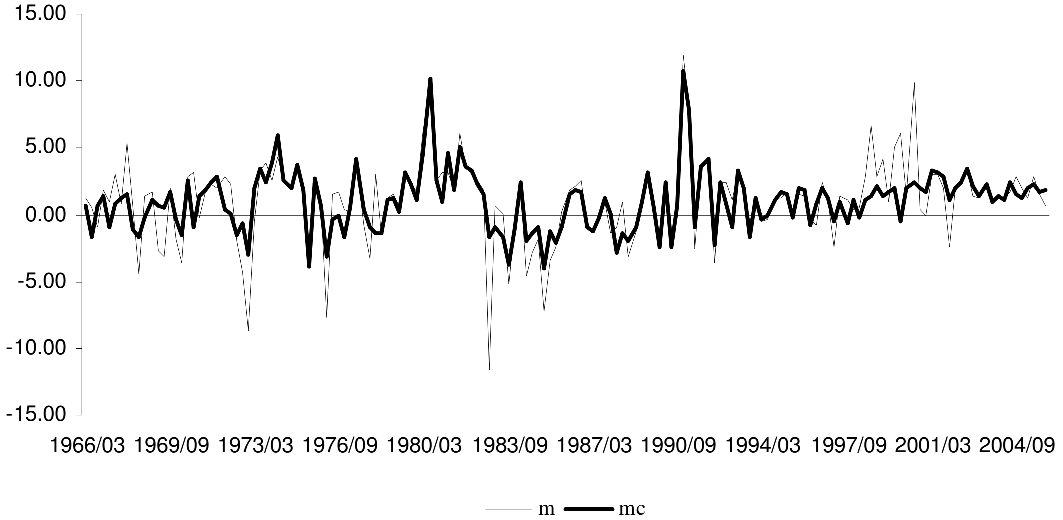
Panel B
(Estimated b_{1t})



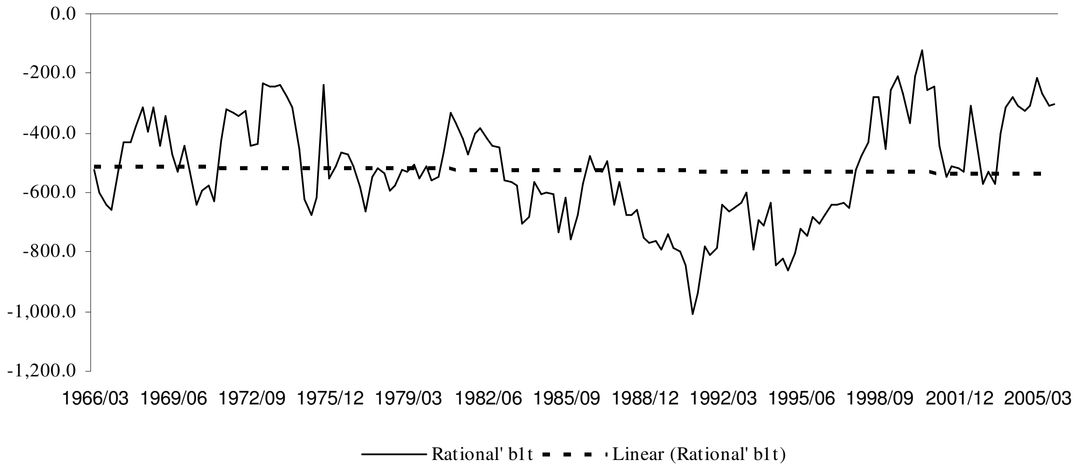
Notes. Panel A of this figure reports the SDF implied by the 2-step regression estimates of the (C)CAPM. Panel B reports the estimate of the conditional b_{1t} parameter for the same model and its linear interpolation (denoted as "linear"). The sample period is 1966-2005.

Figure 2
Sentiment-(C)CAPM

Panel A
(Estimated SDF)



Panel B
(Estimated b_{1t})



Notes. Panel A of this figure reports the SDF implied by the 2-step regression estimates of the Sentiment-(C)CAPM, and its rational component. Panel B reports the estimate of the conditional b_{1t} parameter for the same model and its linear interpolation (denoted as "linear"). The sample period is 1966-2005.

Figure 3
The Sentiment Adjustment

Notes. This figure reports the sentiment component of the SDF implied by the 2-step regression estimates of the Sentiment-(C)CAPM. The sample period is 1966-2005.

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