A Stochastic Discount Factor Volatility Upper Bound in a Mean-Variance-Skewness World: No Good Deal Implications for Multi-Factor Models Estimates

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Comments welcomed

Based on an upper bound on investors' risk aversion, this paper derives a stochastic discount factor (SDF) volatility upper bound that limits attainable maximal Sharpe ratios. The bound and a no-arbitrage condition can be used to rule out "good deals" in incomplete markets without the need to observe investors' optimal portfolios. The usefulness of these restrictions is shown by imposing them on multifactor models non-linear in the market return. Empirical results show that, under the bound, the price of coskewness risk is much reduced and the annualized coskewness risk premium drops, in absolute value, from -2.6 to -1.6 percent. When the test assets include managed portfolios and option-like strategies, conditional versions of a model that nests the Fama and French (1996) 3-factor model and the quadratic market model either violate no-good deal restrictions or are not empirically admissible.

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A Stochastic Discount Factor Volatility Upper Bound in a Mean-Variance-Skewness World: No Good Deal Implications for Multi-Factor Models Estimates

Based on an upper bound on investors' risk aversion, this paper derives a stochastic discount factor (SDF) volatility upper bound that limits attainable maximal Sharpe ratios. The bound and a no-arbitrage condition can be used to rule out "good deals" in incomplete markets without the need to observe investors' optimal portfolios. The usefulness of these restrictions is shown by imposing them on multifactor models non-linear in the market return. Empirical results show that, under the bound, the price of coskewness risk is much reduced and the annualized coskewness risk premium drops, in absolute value, from -2.6 to -1.6 percent. When the test assets include managed portfolios and option-like strategies, conditional versions of a model that nests the Fama and French (1996) 3-factor model and the quadratic market model either violate no-good deal restrictions or are not empirically admissible.

1. Introduction

In recent years, in moving from single factors specifications inspired by the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), to more flexible multi-factor formulations, the asset pricing literature has achieved some success in explaining the cross-section of stock returns[1]. The theoretical motivation of many of the competing specifications is, however, still debated. This is the case, for example, of the Fama and French (1992, 1993) 3-factor model and of models that include momentum, volatility and liquidity-based factors. Moreover, Lewellen, Nagel and Shanken (2006) and, more indirectly, Lewellen and Nagel (2006) warn that apparently successful multi-factor specifications often violate restrictions imposed by the underlying theoretical

[1] Examples of relatively successful multifactor specifications are Fama and French (1992, 1993) 3-factor model, conditional specifications of the CAPM and of the consumption CAPM, as in Lettau and Ludvigson (2001), 3 and 4 moment versions of the CAPM, as in Harvey and Siddique and Dittmar (2002), the closely related quadratic market model estimated by Barone Adesi et al. (2004) and, more recently, models that include momentum, liquidity and volatility factors.

model. For example, the sign of the risk premia estimated by Lettau and Ludvigson (2001) is problematic from the perspective of the conditional consumption CAPM. On a related note, Dittmar (2002) and Post, Levy and Van Vliet (2004) point out that covariance and coskewness risk prices estimated in empirical tests of the 3 and 4 moment CAPM imply a non-concave utility function, to an extent that might be inconsistent with the models being tested. Whenever the theoretical motivation of an empirical factor model is unclear, so that it is difficult to fully restrict a priori its parameters, or when the factor model estimates do not satisfy the restrictions implied by the underlying theoretical model, there is the concrete danger that part of the cross-sectional explanatory power comes from a spuriously volatile stochastic discount factor (henceforth SDF). As explained by Cochrane (2001), a sufficiently volatile SDF can in fact assign arbitrary prices to the idiosyncratic portion of the test asset payoffs and thus generate arbitrary small (in-sample) pricing errors, even if relevant factors are omitted and the model is grossly mis-specified[2]. See Wang and Zhang (2005) for similar considerations on the need to rule out arbitrage and thus a non-positive SDF.

This paper addresses the problem of bounding from above SDF volatility. The identification of a sensible SDF volatility upper bound is both a theoretical and an empirical problem. To address it, I seek guidance from the minimum variance SDF that prices the test payoffs, from long-run SDF volatility estimates and from the Sharpe ratio (henceforth SR) achievable using a combination of buy-and-hold and option strategies. More importantly, I derive a SDF volatility upper bound that applies, for a given upper

[2] There is also the possibility that part of the differences in the average historical returns across stocks do not reflect genuine differences in expected returns and are due instead to in-sample good or bad luck, i.e. a Peso problem as defined for instance in Evans (1996). In this case, the SDF volatility required to price the cross-section of ex-post return, given average ex-post returns, would be different from the volatility of the SDF that prices the test asset payoffs (ex-ante).

bound on investors' relative risk aversion, under a fairly broad family of utility functions and mild assumptions about the distribution of returns. This bound is a generalization of the bound developed by Ross (2005). To demonstrate the usefulness of the bound, I consider SDF models that contain quadratic terms in the market return. Because of their non-linearity, these specifications are particularly prone to excessive SDF volatility and thus they represent ideal candidates for a constrained estimation[3]. In particular, I study the unconditional implications for the cross-section of stock returns of restricted and unrestricted versions of a model that nests the Fama and French (1993) 3-factor model and the quadratic market model used by Barone Adesi, Gagliardini and Urga (2004) and Potì (2005), with and without positivity and volatility constraints on the corresponding SDF. The first constraint rules out arbitrage opportunities, while the second rules out unduly high Sharpe ratios. Together, these restrictions mitigate the danger of over-fitting the cross-section of excess-returns. At the most abstract level, this method can be interpreted as an application of Ross' (1978) principle of linear pricing on an approximate basis, in an arbitrage and near-arbitrage free setting[4]. The estimation is conducted using both robust 2-pass regressions and more efficient GMM estimators. I also show that expanding the set of test asset payoffs to include payoffs from option-like and managed strategies, minimizing Hansen and Jagannathan (1997) pricing error and allowing for an intercept in second pass regressions represent alternative, yet not fully equivalent, methods to guard against spurious SDF volatility.

[3] Moreover, coskewness likely captures the exposure to a number of factors typically included in empirical asset pricing studies. For example, there is evidence that coskewness might proxy for exposure to the momentum effect documented by Jegadeesh and Titman (1993), see for example Potì (2005) or the evidence provided by Hong, Lim, and Stein (2000), who argue that momentum is asymmetric and is stronger in declining stocks than in rising stocks. Similar considerations suggest that coskewness might proxy for exposure to the liquidity factor of Pastor and Stambaugh (2003).

[4] The idea of specifying an assumption about the maximum available SR to rule out "approximate arbitrage opportunities", when pricing a limited universe of stocks on the basis of the APT, was already advocated by Bernardo and Ledoit (2000) who refer to Ledoit's (1995) unpublished Ph.D. thesis and it traces back to Ross (1976, 1978).

In the next Section, I present background analytical results on stochastic discount factor pricing and I introduce and compute the SDF volatility upper bound. Section 3 presents the dataset. Section 4 outlines the estimation methodology and reports the important empirical results. In Section 5, I expand the set of test asset payoffs, I estimate using conditioning information and I include an intercept in second pass regressions. The final Section summarizes the main findings and draws together the conclusions.

2. Asset Pricing and SDF Volatility

A well known theorem, credited to Harrison and Kreps (1979), says that, given free portfolio formation and the law of one price, a stochastic discount factor[5] m_{t+1} that prices any set of traded payoffs exists. This process satisfies the following condition for any payoff x_{t+1} and payoff price p_t:

$$p_t = E_t(m_{t+1} x_{t+1}) \tag{1}$$

Here, the expectation is taken conditional on the available information set. Under the additional assumption of no arbitrage, m_{t+1} must be positive[6]. Since the price of excess returns $r_{i,t+1}$ is by definition zero, (1) can be rewritten as follows:

$$0 = E_t(m_{t+1} r_{i,t+1}) \tag{2}$$

[5] See also Hansen and Richard (1987) for a seminal reference on stochastic discount factor pricing.
[6] If the set of the assets being priced spans all possible payoffs, m_{t+1} is unique. Instead, if the payoffs on the priced assets are only a subset of the universe of payoffs, there is an infinite choice of processes m_{t+1} that satisfy (1). These processes share the same projection on the priced payoff space. Trivially, any linear combination of these processes prices the assets.

Since the absolute value of correlation is bounded from above by 1, (2) implies that, under no-arbitrage, admissible spreads in expected excess returns across assets are proportional to SDF volatility[7]:

$$\left|E_t(r_{i,t+1})\right| \le \left[(1+R_{f,t})\sigma_t(r_{i,t+1})\right] \sigma_t(m_{t+1}) \qquad (3)$$

On a related note, Bakshi, Chen and Hjalmarsson (2004) illustrate a one-to-one correspondence between SDF volatility and the distance that separates risk-neutral and objective probability measures. Ross (2005) proves that risk-averse investors prefer volatile SDFs. More specifically, Proposition 1 in Ross (2005) demonstrates that, in a complete market and given a family of SDFs with the same mean but different volatility, the expectation of a concave utility function is uniformly larger if the payoffs are priced by a more volatile SDF. Cochrane and Saà Requeio (2000) and Cochrane (2001) introduced the term 'good deals' to denote desirable investment opportunities, i.e. arbitrage opportunities and investment opportunities that offer a large reward for risk. Similarly, Cerný and Hodges (2000, 2001) define good deals as desirable investment opportunities that have zero or negative cost.

SDF Volatility Upper Bound

An implication of Ross' (2005) Proposition 1 is that, in a complete market and from the point of view of a risk-averse investor, if the SDF is either not strictly positive or more

[7] Noting that (2) must hold for any SDF that prices the assets, including the minimum-variance SDF, and rearranging, also yields the well-known Hansen and Jagannathan (1991) lower bound on SDF volatility.

volatile than the investor's inter-temporal marginal rate of substitution (henceforth IMRS), but the SDF and the IMRS have the same mean, the investment opportunity set offers good deals[8]. Thus, a natural way of bounding from above SDF volatility is to first find an upper bound to the volatility of the investors' IMRS and then extend this bound to the SDF by ruling out good deals. An investor's IMRS is simply her marginal utility growth. Thus, when studying an investor's valuation of excess returns, i.e. zero-cost payoffs, we can use marginal utility (henceforth MU) in place of the IMRS. This is a convenient simplification that we will adopt here.

To bound from above an investor's MU, suppose that her preferences can be described by a continuous, smooth, non-polynomial[9], time-invariant, concave utility function $U(W_U)$ defined over her wealth W_U. Further, suppose that these preferences conform to standard risk aversion, as defined by Kimball (1993), i.e. assume non satiation (NA), risk aversion (RA) and non increasing absolute risk aversion (NIARA). Denote by $R_{U,t+1}$ the return on the investor's portfolio and by m_{Ut+1} her MU. Consider next a monotonic, concave transformation $V = G(U)$ of the investor's utility function and denote by RRA_V its relative risk aversion coefficient. By Pratt's (1964) Risk Aversion Theorem, U is bounded in risk aversion by V. Also, assuming that a risk-free asset exists and imposing

[8] Since an investor's price of risk is proportional to the volatility of his IMRS (for a given the mean of the latter), while the market price of risk is proportional to the volatility of the minimum-variance SDF m_{t+1}^* (again, given its mean), good deals are investment opportunities characterized by a market price of risk larger than an investor's required price of risk.

[9] As argued by Levy (1969) and Tsiang (1972), finite order polynomial utility functions are unsuitable to model the preferences of a risk adverse investor. Duly restricted third order Taylor expansions of admissible non-polynomial utility functions can be used instead. Moreover, a non-polynomial utility function can be in principle unbounded. In turn, based on Theorem 16 of Cerný and Hodges (2000), this is an important requirement for being able to extend the implications of absence of good deals from a complete to an incomplete market, i.e. for the absence of 'good deals' in complete markets to imply the existence of a (linear) pricing functional such that the no-good deal requirement also holds for any subset of the assets.

no-arbitrage, the risk-free return identifies the mean of MU. Thus, by Ross' (2005) Proposition 3[10]:

$$\sigma^2(m_{Ut+1}(R_{Ut+1})) \leq \sigma^2(m_{Vt+1}(R_{Ut+1})) \tag{4a}$$

Next, to extend this MU volatility bound to SDF volatility, assume that the investor can borrow and lend unlimitedly at the risk-free rate and that the traded and non-traded payoffs available to her span the entire payoff space, i.e. the space of all possible payoffs in the economy. If the market is incomplete, it is useful for our purposes to 'complete' it by imagining that the price of the non-traded payoffs corresponds to the investor's marginal valuation of these payoffs. This is a necessary condition for her to be the representative investor. If we rule out good-deals, then, Ross' (2005) Proposition 1 implies that the volatility of the SDF that prices the available payoffs cannot exceed the volatility of the investor's MU. Thus, (4a) becomes an upper bound to SDF volatility. This bound requires knowledge of the investor's efficient portfolio R_U. Since the latter might be unobservable, however, we would like to find a bound defined over an observable portfolio. To this end, consider the following Proposition:

Proposition 1: the expected utility of a risk-averse investor decreases in the mean and volatility of her MU.

Proof: By assumption, utility $U(W_{t+1})$ is concave, monotonically increasing in wealth W_{t+1} and its shape is consistent with NIARA. Therefore, marginal utility $U'(W_{t+1})$ is positive, convex and

[10] Proposition 3, in Ross (2005), states that, if the concave utility function *U* is bounded from above in risk aversion by the utility function *V*, the second moment of the IMRS of the former is bounded from

monotonically decreasing in wealth. Since the square is a monotone convex transformation over the positive range of a variable, $U'(W_{t+1})^2$ is also convex and monotonically decreasing in wealth. Utility and the square of marginal utility display the opposite behaviour in the same variable. Therefore, the former must be decreasing in the latter and this must be also true of their expectations. From elementary statistics, it is well known that $E(X^2) = E(X)^2 + \sigma^2(X)$ for every random variable *X*. Thus,

$$E[U'(W_{t+1})^2] = E[U'(W_{t+1})]^2 + \sigma^2[U'(W_{t+1})]$$

This implies that, since expected utility is decreasing in $E[U'(W_{t+1})^2]$, it must also be decreasing in the mean and volatility of $U'(W_{t+1})$. QED

Since *V* = *G*(*U*) is a monotonic, concave transformation of *U* and the latter is concave, *V* is also concave. Moreover, the return on the risk-free asset identifies the mean of the MU. Then, recalling that R_{Ut+1} is the investor's efficient portfolio, Proposition 1 implies

$$\sigma^2(m_{Vt+1}(R_{Ut+1})) \leq \sigma^2(m_{Vt+1}(R_{t+1})) \qquad \forall\ R_{t+1} \tag{4b}$$

Here, R_{t+1} is the return on any strategy that is not more desirable than $R_{U,t+1}$. We could take R_{t+1} to be the return on a diversified stock market index, say $R_{m,t+1}$. We would then have $\sigma^2(m_{Vt+1}(R_{Ut+1})) \leq \sigma^2(m_{Vt+1}(R_{mt+1})$ and thus, using (4a),

$$\sigma^2(m_{Ut+1}(R_{Ut+1})) \leq \sigma^2(m_{Vt+1}(R_{mt+1})) \tag{4c}$$

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above by the second moment of the IMRS of the latter. For a given mean of the IMRS, (4a) follows.

Clearly, if we were able to identify a strategy more desirable than buying and holding the stock market portfolio, we would have a sharper, i.e. lower, upper bound on the volatility of the investor's MU. The bound in (4c) is not usable yet as the functional form of m_{Vt+1} needs to be specified. Appendix A approximates the latter using a third order Taylor expansion of *V* and derives bounds on its coefficients. Using these, we can write

$$\sigma^2(m_{Vt+1}(R_{mt+1})) \leq b_{V1,\min}^2 \sigma^2(R_{mt+1}) + b_{V2,\max}^2 \sigma^2(R_{mt+1}^2) + b_{V1,\min} b_{V2,\max} Cov\ (R_{mt+1}, R_{mt+1}^2) \tag{4d}$$

Here, $b_{V1,\min} \equiv -RRA_V^{\max}$, $b_{V2,\max} \equiv \frac{1}{2}\frac{RRA_V^{\max}}{R_{mt+1}^{\max}}$, while $RRA_V^{\max}$ and $R_{mt+1}^{\max}$ denote the maximum values taken by RRA_V and R_{mt+1}, respectively, over the range of the possible realizations of the latter. Combining (4c) and (4d),

$$\sigma^2(m_{Ut+1}(R_{Ut+1})) \leq b_{V1,\min}^2 \sigma^2(R_{mt+1}) + b_{V2,\max}^2 \sigma^2(R_{mt+1}^2) + b_{V1,\min} b_{V2,\max} Cov\ (R_{mt+1}, R_{mt+1}^2) \tag{4e}$$

This bound applies under any distribution of returns for which moments of at least the first four orders exist (and are finite). It is therefore a generalization of the result provided by Ross (2005) that instead applies only when utility is quadratic or returns are normally distributed. The bound is based on an upper bound to the risk aversion of a hypothetical investor that holds a portfolio at least as desirable, given concave utility, as a diversified stock portfolio. It does not matter whether the investor with bounded risk aversion is the representative investor. It is important, however, that she is in the position of acting as the representative investor if good deals, i.e. either arbitrage opportunities or investments that offer a large enough SR, become available. To this end, the assumption that she has access to all available payoffs is crucial. If this was not the case, i.e. if her available payoffs did not 'complete' the payoff space of all other investors who are in a position of influencing asset prices, the volatility of her MU would not bound from above the SR of all investment opportunities, but only of a subset of the latter. On the other hand, a sufficient condition for Ross' (2005) wealthy individual (or pool of wealthy individuals) that hold the stock market portfolio to act as the representative investor is that the stock market is 'price-complete', in the sense that no non-traded payoff serves any net hedging demand. In this case, as argued by Ross (2005), the price of any new payoff is uniquely determined by the (minimum variance) SDF that prices the traded payoffs. Then, an upper bound on the relative risk aversion of the wealthy individual (or pool of individuals) that at the margin holds the stock market portfolio becomes a suitable risk aversion upper bound in (4d), i.e. it becomes a suitable candidate for $RRA_V^{\max}$. This, however, is a special case.

Computing the Bound

A study by Meyer and Meyer (2005) has recently provided a comprehensive re-evaluation of the hitherto rather scattered empirical evidence on investors' risk aversion. They show that relative risk aversion estimates reported by the extant literature are less heterogeneous and extreme if one takes into account measurement issues and the outcome variable with respect to which each study defines risk aversion. Using returns on stock investments as the outcome variable, calculations by Meyer and Meyer (2005) show that the RRA coefficient in the classical Friend and Blume's (1975) study of household asset allocation choices ranges between 6.4 and 2.0, and decreases in investors' wealth. Using returns on the investors' overall wealth, including real estate and a measure of human capital, the RRA estimate ranges between 3.0 and 2.4. The same calculations show[11] that the RRA implied by Basky et al. (1997) survey data ranges between 0.8 and 1.6. Importantly, these estimates are never much higher and, instead, are usually considerably lower than 5, i.e. the upper bound suggested by Ross (2005).

I compute the SDF volatility upper bounds, based on (4e), under two different upper bounds on RRA. The first bound is 5 and corresponds to the bound suggested by Ross (2005). The second bound is 6.4 and corresponds to the RRA coefficient of the most risk-averse cohort of investors in Friend and Blume (1975) study, as re-calculated by Meyer and Meyer (2005). The 5 and 6.4 upper bounds on RRA imply that the investor would be willing to pay no more than 10 and 12.8 percent per annum, respectively, to avoid a 20 percent volatility of her wealth (i.e., about the 1926-2002 annualized volatility of the S&P 500 index). By introspection, these are arguably large amounts. The different assumptions used in the computation of the bounds, reported in Table 1, reflect sample moments estimated over the period 1927-2005 and portions thereof[12], and market highs over the same periods[13].

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[11] Meyer and Meyer (2005) calculate somewhat higher values based on estimates provided by studies of the equity premium puzzle. Since these estimates are backed out parametrically from estimates of a particular asset pricing model, often based on a narrow definition of the market portfolio, they are of no interest for the purpose of computing the SDF volatility bound in (4e). Moreover, their use would imply a circular argument.

[12] This implies the assumption that the representative investor's expectation of the factor variance-covariance matrix (and thus of market volatility and skewness) is rational and that there is no small sample problem, i.e. 'Peso problem', in the estimation of factor second moments. The sampling error of returns second and third moments is much lower than the sampling error of first moments. Thus the former are much easier to estimate than the latter. As a consequence, it is not unreasonable to rule out

3. Data

I use monthly data from 1926 to 2002 on industry-sorted portfolios formed following Fama and French (1992). In particular, I use data constructed sorting stocks of the Centre for Research on Security Prices (CRSP) database into an overall market portfolio, and 17 and 30 industry portfolios[14]. I also use quarterly returns on the 3-month US Government Treasury Bill as a proxy for the risk free rate. Finally, I construct time series of excess returns on at-the-money 3-month option contracts written on each one of the 17 industry-sorted portfolios and held until expiry. The options are valued according to the Black and Scholes (1973) model, using quarterly conditional volatilities as inputs in the Black and Scholes call pricing formula. These volatilities are estimated using the type of GARCH(1,1) model specified by Glosten, Jagannathan and Runkle (1993), that allows for an asymmetric reaction of the conditional variance estimates to return innovations of different signs.

4. Factor Model Estimates

The parameters of the SDF that prices a vector of n test asset payoff x_{t+1}, and that satisfies a no arbitrage condition and a volatility upper bound, can be estimated by solving the following problem:

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errors in forming returns second and third moment estimates while allowing for errors in expected first moments (that might motivate imposing the SDF volatility bound in the first place).

[13] The reported bounds are not far from the bound computed by Ross (2005), i.e. $\sigma(m_{t+1}) \leq 96$ percent, using $RRA_V \leq 5$ and 20 percent annual market volatility, corresponding to the S&P 500 volatility over the period 1926-2002. Using the unconditional market volatility estimate over the period 1952-2002, Ross' (2005) upper bound would be 74 percent.

[14] I thank K. French for making this data publicly available for download.

$$\min_{\{m\}} \quad g_T' W_{n(1+k)\times n(1+k)} g_T$$
$$s.t. \;\; m_{t+1} \geq 0, \;\; \sigma_T^2(m_{t+1}) \leq A \tag{5}$$

Where,

$$g_T = E_T\left[(m_{t+1}x_{t+1} - p_t) \otimes z_t\right] \tag{6}$$

Here, $E_T(.)$ denotes a sample average, i.e. an arithmetic average over a sample of T observations, and z_t is a vector of k instruments. When pricing excess returns, $x_{t+1} = r_{i,t+1}$ and $p_1 = 0$. The elements of the $n\times 1$ vector g_T correspond to the moment conditions implied by the factor pricing model in (1) and thus they can be interpreted as pricing errors sample averages. Under the usual ergodicity assumption, the latter are consistent estimates of the unconditional expectations of the corresponding moment conditions. The $n\times n$ matrix W is a weighting matrix for the moment conditions. In Appendix C, I briefly discuss the choice of the weighting matrix and the conditions under which the solution to the problem in (5) yields GMM, OLS and GLS estimates. Factor models specify m_{t+1} as a linear function of a set of factors f_{t+1}:

$$m_{t+1} = a_t + b_t' f_{t+1} \tag{7}$$

Solving the problem in (5) thus entails searching for the values of the elements of b that satisfy the positivity and volatility bounds[15] and minimize an appropriate pricing error metric, defined by a suitable weighting matrix. To model the SDF in a flexible manner and to allow for the possibility that investors' preferences be defined over systematic second and third moments, I include the market return $R_{m,t+1}$ and its square amongst the factors in (7). Letting $f_{t+1} = \begin{bmatrix} R_{m,t+1} & R^2_{m,t+1} \end{bmatrix}$ yields a quadratic market model (QMFM). Imposing the existence of a conditionally risk free rate $R_{f,t}$, and following Barone Adesi, Gagliardini and Urga (2004) and Potì (2005), the QMFM implies $m_{t+1} = 1 + b_{1,t} r_{m,t+1} + b_{2,t} q_{m,t+1}$. Here, $r_{m,t+1} = R_{m,t+1} - R_{f,t}$ and $q_{m,t+1} = R^2_{m,t+1} - R_{f,t}$ can be seen as a new set of factors. Restricting $b_{2,t}$ to equal zero yields a linear market factor model (LMFM). Writing the SDF in (7) as a linear function of size and book-to-market factor mimicking portfolios yields the the Fama and French (1993, 1995) 3-factor model (henceforth FFM). Combining the Fama and French (1993, 1995) factors and $q_{m,t+1}$ yields a model that nests the QMFM and FFM. I call this specification the QMFM-FFM model. In the ensuing empirical analysis, I will consider the following representations of the restrictions that (2) and (7) impose on the cross-section of expected returns[16]:

$$E_t(r_{i,t+1}) = -Cov_t(f_{t+1}, r_{i,t+1})b_t \tag{8}$$

$$E_t(r_{i,t+1}) = \beta'_{i,t}\lambda_t \tag{9}$$

Where,

$$E_t(m_{t+1}) = a_t + b'_t E_t(f_{t+1}) \cong 1 \tag{10}$$

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[15] The approach outlined in (5) can be seen as a type of mixed estimation, see Theil (1971) for a seminal reference, that uses the volatility bound as a prior.

[16] In (10), for convenience, I set the mean of m_{t+1} equal to 1. This is legitimate because, as explained by Cochrane (2001), excess returns do not identify the mean of the SDF and thus the latter can be set equal

The elements of $-b_t$ can be seen as the factor risk prices and $\beta_{i,t} = Var_t(f_{t+1})^{-1} Cov_t(f_{t+1}, r_{i,t+1})$ is a vector of factor loadings of the regression of asset i on the factors. The elements of $\lambda_t \cong -Var_t(f_{t+1})b_t$ are factor risk premia. Following a widely used terminology, I will refer to (8) and (9) as covariance and beta-pricing representations, respectively.

Unconstrained Estimates

I first estimate[17] the LMFM, QMFM, FFM and QMFM-FFM models with fixed b. I also experiment with various combinations of the factors. I estimate these models using robust 2-pass regressions without intercept in the second pass. As explained in Appendix C, a 2-pass regression is equivalent to minimizing (5) using the identity matrix to weight the moment conditions, i.e. setting $W = I$, while a zero intercept in second pass regressions is equivalent to imposing no SDF volatility bound. The sample period is 1952-2002. I estimate OLS standard errors corrected to take cross-sectional error correlation into account and, following Shanken (1992), for the fact that factor loadings are estimated. For each model, I also estimate the volatility $\sigma(m_{t+1})$ of the

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to an arbitrary value. This, however, is strictly true only as long as the risk free rate is not unrealistically high.

[17] Since, as shown by Jagannathan and Wang (2002), the Beta-method and the SDF-method are equivalent in terms of consistency and asymptotic efficiency, I estimate the LMFM, QMFM and FFM following the former because of its greater simplicity. This way, I directly estimate the parameters of the beta-pricing representation of these models. In a 2-step procedure, I first regress the time series of the 30 industry portfolios and size and book-to-market sorted portfolios excess returns on the factors allowing for an intercept in the regression equations. This way I estimate the factor loadings β. I then estimate the risk premia λ using a cross sectional regression of the average portfolio returns on the factor loadings estimated in the first step, without an intercept term. I then obtain the Covariance-pricing representation of these estimates. This procedure yields exactly the same estimates of the SDF as (5) under the appropriate choice of the weighting matrix and constraints (and without instruments apart from a constant). In particular, this is the case if we set the weighting matrix equal to the identity matrix, we remove the positivity bound and we impose the constraint of zero pricing error on the equally-weighted combination of the industry portfolios, i.e. $E_T(m_{t+1}r_{EW,t+1}{}^2) = 0$. Removing the volatility bound is equivalent to estimate the second pass regression without intercept.

corresponding SDF. This is done by taking the sample standard deviation of m_{t+1}, given the sample realizations of the factors and the point estimates $\hat{b}$ of the model parameters or, equivalently, by computing $\sigma(m_{t+1}) = \hat{b}' Var(f_{t+1})\hat{b}$, where $Var(f_{t+1})$ is the sample variance-covariance matrix of the factors.

The unconstrained estimates for the 30 industry-sorted portfolios are reported in Table 2. The QMFM displays a larger explanatory power than the FFM specification. The sign of the market risk premium is positive, in accordance with the notion that the typical investor is averse to systematic stock market risk. The coefficient of the squared market return polynomial factor is negative, thus satisfying a necessary condition for decreasing absolute risk aversion (DARA) and preference for skewness. Adding the squared market return factor increases the cross sectional explanatory power, while preserving the positive sign of the market risk premium. Because there is considerable cross-sectional dispersion, industry returns are notoriously difficult to fit. Thus, relatively low coefficients of cross-sectional determination should not surprise and are in line with the estimates reported by Harvey and Siddique (2000). The SDF of the QMFM, however, violates the no-arbitrage requirement as it takes negative values over a range of market return realizations, as shown in Figure 1, and thus it does not always assign a positive price to strictly non-negative payoffs. The estimated SDF is also very volatile, especially in the case of the QMFM. The volatility of the latter appreciably exceeds the upper bounds reported in Table 1. Perhaps surprisingly, it also largely exceeds the volatility of the minimum variance SDF[18] that prices by construction the test assets, shown in Figure 2. As also shown in Table 2, moreover, estimated SDF volatility is considerably lower over the longer 1926-2002 sample period[19]. The 1952-2002 point estimates of the covariance and coskewness risk prices imply a SDF that is increasing over a range of market wealth.

Constrained Estimates

Next, I estimate the QMFM and, for comparison, the FFM and the QMFM+FFM models by solving (5) under positivity and volatility constraints on the SDF. I experiment with three different volatility upper bounds, i.e. 50, 75 and 100 percent, by setting in (5) A = 0.25, 0.75 and 1.00. The 50 percent bound is broadly in line with the 1926-2002 SDF volatility estimates and, based on (4e), it implies an upper bound of 2.5 on the relative risk aversion of the marginal investor. The other values, i.e. 75 and 100 percent, broadly correspond to the values reported in Table 1, and thus to relative risk aversion upper bounds of 5 and 6.4, respectively. They are also very close to the volatility of the minimum-variance SDF, plotted in Figure 1, that prices by construction the 17 and 30 industry portfolios, respectively, given a SDF mean consistent with the sample risk-free rate. The estimation is conducted by iterated GMM[20], and thus using

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[18] For example, the minimum variance SDF that prices the 30 industry portfolios displays an annualized volatility of about 86 percent, assuming a 94.8 percent mean (consistent with the sample mean of the risk-free rate, about 5.2 percent). This is well below the volatility of the estimated SDF reported in Table 2.

[19] To assess the sampling variation in the SDF volatility and thus the sampling error of σ(m), I conduct a simplified 'bootstrap' experiment, see Efron (1979) for a seminal reference. I generate 500 simulations of the factor and industry portfolio time series sampling with replacement three consecutive observations at a time from their 1952-2002 time series and I re-estimate (19) at each simulation, with no positivity and volatility bound but with $W = I$ and the constraint $E_T(m_{t+1} r_{EW\,t+1}) = E_T[p_t(r_{EW\,t+1})]$ to obtain second pass regression OLS estimates. The average annualized SDF volatility across the simulations is 75.43 with a very large sampling variation. The standard deviation of the annualized simulated SDF volatility is 36.22 percent. The annualized SDF volatility 5th percentile is 0.27 whereas its 95th percentile is 1.45.

[20] Because this study focuses on the cross-section of excess-returns rather than on the equity market premium, I do not impose $E_T(m_{t+1} f_{t+1}) = E_T[p_t(f_{t+1})]$. This choice is also motivated by the circumstance that the estimation of the moments of the market portfolio distribution is subject to the same sampling uncertainty as the pricing of the industry portfolios that constitute the test assets. Therefore, from an approximate linear pricing perspective, there is no strong reason to attribute the market portfolio a special status.

the inverse of a consistent estimate of (C1), see Appendix C, as the weighting matrix of the moment conditions, and without instruments, i.e. by setting z_t equal to 1. I report in Table 3 the risk prices estimates, i.e. the elements of $\hat{b}$, and Hansen's (1982) TJ_T statistics. To facilitate the comparison between the estimated models, the same statistics are also re-calculated using a pre-specified weighting matrix. The latter is the consistent iterated GMM estimate of the S^{-1} matrix with zero lags[21], i.e. $\hat{S} = E_T\left[u_t(b)u_t(b)'\right]$, of the QMFM+FFM model under a 100 percent SDF volatility bound. Finally, Table 3 also reports Hansen and Jagannathan (1997) distances, which are less sensitive to SDF volatility. In Table 4, I report factor risk premia estimates and pseudo R^2 coefficients, i.e. coefficients of determination computed as the squared cross-sectional correlation between average returns and the portion of the latter explained by the estimated model.

Under the volatility bounds, the coskewness risk price estimates are substantially smaller and the corresponding risk premium is also smaller in absolute value. For example, under the 75 percent volatility bound, the coskewness risk price is almost cut in half relative to its unconstrained estimate and the coskewness risk premium drops in absolute value from -0.66 to -0.40 percent per quarter (from -2.64 to -1.6 percent per annum, i.e. about 1 percent less in absolute value). The reduction in the covariance risk price is equally impressive and it leads to a covariance risk premium over 3 percent lower on an annualized basis (from 1.90 to 1.11 percent per quarter and from 7.6 to 4.44 percent per annum). These are sizeable differences from a capital budgeting and valuation perspective. Interestingly, however, Hansen's (1982) TJ_T statistics are not

[21] This specification of the S matrix corresponds to a null hypothesis that the pricing errors are unpredictable. It neglects possible serial auto-correlation and cross-correlation of the pricing errors and thus it is sub-efficient. This simplification however is more robust to mispecification errors and, while it

significantly larger under tighter volatility bounds and Hansen and Jagannathan (1997) distances are actually lower under the 75 percent bound than under the 100 percent bound. This suggests that the former is indeed the appropriate SDF volatility upper bound[22].

5. Further Analysis and Robustness Checks

To check on the robustness of the estimates obtained under the SDF volatility upper bound and to gain a deeper insight into the role played by SDF volatility in unrestricted estimates, I first re-estimate the SDF using an augmented set of test asset pay-offs. The augmentation, designed to span a larger set of states of the World, is achieved by using portfolios managed on the basis of conditioning information conveyed by Lettau and Ludvigson (2001) consumption-wealth ratio (cay_{t-1}) and by including the constructed industry option excess-returns. The SDF volatility implied by unconstrained QMFM and QMFM-FFM estimates generally decreases when the augmented set of test asset payoffs is used, as shown in Table 5. This is an important finding and adds weight to the concern that unconstrained estimates of the QMFM and QMFM-FFM fit the cross-section of industry portfolios partly thanks to a spuriously volatile SDF, that prices relatively well the bulk of their payoffs but prices very poorly the residual portion, likely captured by the constructed option payoffs. Moreover, the volatility of the SDF is noticeably lower when the QMFM is estimated using Hansen and Jagannathan (1997) weighting matrix. This is not surprising, as it is well known that Hansen (1982)

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likely does not affect much the size of the test because returns are not very auto-correlated at this frequency, it increases its power. See Cochrane (2001) for a discussion.

[22] In the case of the FFM specification, volatility bounds higher than 67.2 percent are slack, because this is the maximum amount of SDF volatility that this specification can generate when fitting the cross-section of industry returns.

weighting matrix rewards volatile pricing errors and thus volatile SDFs. This consideration, however, adds further weight to the concern that SDF volatility, in unconstrained estimates of the QMFM, is spuriously high. Appendix B presents additional arguments that suggest that the SR implied by unrestricted estimates is unrealistic, by showing how difficult it is to generate such a high SR using option contracts, under realistic assumptions about implied volatility.

I also consider more flexible conditional specifications of the SDF. In particular, I specify a conditional version of the FFM model (C-FFM) and a model that is conditionally quadratic in a market portfolio augmented by the return on human capital, i.e. labour income. I call the latter the conditional Quadratic Factor Model (C-QMFM). Such a specification is very close to the conditional quadratic SDF estimated by Dittmar (2002). There is the concern that using too many conditioning variables in modelling the dynamics of the SDF parameter would leave too few degrees of freedom in estimation. Parsimony is therefore important and I consequently use only two predictive variables to capture the time-variation in risk prices. These variables are the consumption-wealth ratio cay_t estimated by Lettau and Ludvigson (2001) and the spread s_t between 3 month and 1 month T-Bill rates. As shown in Table 6, the former is the variable that displays most predictive power and, more importantly, its explanatory power increases with the horizon, due to its persistence. Thus, while it is little correlated with contemporaneous returns, helping avoid identification problems, it captures expected returns because it is highly correlated with future returns. The estimation results are reported in Table 7. In unrestricted estimates, only the C-QMFM is not rejected based on the asymptotic distribution of Hansen and Jagannathan (1997) distance. Under no-arbitrage, however, this specification too is rejected at the 5 percent level. Moreover, even under the no-arbitrage restriction, SDF volatility largely exceeds the bounds in Table 1. These results imply that, while the QMFM and, to some degree, the FFM model satisfactorily price the cross-section of excess returns on static strategies, they perform unsatisfactorily when pricing payoffs from dynamic strategies and strategies that use conditioning information. This failure leaves considerable scope for future research[23].

I finally re-estimate without imposing any explicit SDF volatility bound but allowing for an intercept in second pass regressions. This is equivalent to restricting the volatility of the estimated SDF to the SR not explained by the intercept in time series regressions of the test asset payoffs on the factors. In fact, as reported in Table 8, the SDF volatility is considerably lower than in unrestricted estimates, i.e. than in the estimates reported in Table 2 and 3. The volatility of the estimated SDF is 91.2 and 85.5 percent in the case of the QMFM and QMFM-FFM, respectively. These values are just above the SDF upper bound for $RRA_V = 5$ and even below the SDF upper bound for $RRA_V = 6.4$. Since second pass estimates with intercept are more robust to mispecification errors, these results demonstrate the usefulness of bounding from above SDF volatility in making inferences about factor risk prices. Two-pass regressions are, however, a special case of (5), as they imply relatively simple specifications of the weighting matrix W, i.e. either

[23] The amount of SDF volatility required to price these strategies is puzzling. The estimated SDF volatility corresponding to the only empirically admissible model, i.e. the conditional version of the QMFM, is 224 per cent per annum. A drop in SDF to 154 percent per annum, under the no arbitrage restriction, is enough to render the model empirically inadmissible. Yet these values are largely below the SDF volatility upper bounds in Table 1. While it cannot be ruled out that the bounds are too conservative, a realistic possibility is that the managed portfolios correspond to unfeasible strategies, i.e. strategies with unfeasibly high SRs. Appendix B shows how difficult it is to construct strategies that generate reliably high SRs, even resorting to a likely good candidate for this purpose, i.e. selling options. On a related note, Luttmer (1996) shows how even modest proportional transaction costs, short sales restrictions and margin requirements considerably lower the mean-variance SDF frontier. These considerations, based on the minimum-variance SDF frontier for economies with frictions derived by Hansen, Heaton and Luttmer (1995), are in sharp contrast with the common practice of tightening SDF volatility lower bounds

the identity matrix in the case of OLS or a slightly more complex matrix in GLS estimates. Moreover, it is impossible to impose no-arbitrage in 2-pass regression estimates. From this perspective, estimating (5) under SDF positivity and volatility restrictions, can be interpreted as a flexible way of retaining the robustness of 2-pass regressions, while being able to impose no-arbitrage and other restrictions based on a priori information, such as the bound on SDF volatility in (4e).

6. Concluding Remarks, Main Findings and Future Research

The idea of employing volatility restrictions on the SDF in approximate linear pricing dates back to Ross (1976). This idea, however, did not see widespread application in the literature on the cross section of stock returns, yet it was not forgotten in the literature on contingent claims, because it seemed that volatility bounds on the SDF cannot be tight enough to be useful when pricing stocks. This is not surprising, as for long time the main problem of the empirical literature has been the lack of volatility of the SDF implied by the prevailing asset pricing theory. For example, the CAPM and its extensions typically struggle to reach the Hansen and Jagannathan (1991) SDF volatility lower bound.

Empirically, unrestricted estimates of specifications quadratic in the market return imply SDF volatility in excess of the level corresponding to a sensible upper bound on investors' relative risk aversion, especially when the test assets include managed portfolios and option-like strategies. Restrictions on SDF volatility lead to substantially different risk price point estimates. For example, with 75 percent annualized SDF

augmenting the payoff space with portfolios managed on the basis of conditioning information or, more volatility, the price of coskewness risk is cut in half relative to unrestricted estimates, and the corresponding annualized coskewness premium drops, in absolute value, from -2.6 to -1.6 per cent.

This paper is the first to show that sensible upper bounds on volatility, together with a positivity constraint, are useful in sharpening the pricing accuracy of non-linear SDF specifications. This result is encouraging as more flexible multi-factor specifications become popular, because it suggests that future research can fruitfully explore additional ways to restrict the moments of the estimated SDF without having to impose the structure implied by a full asset pricing model. Other applications of the SDF volatility upper bound are possible. For example, it may be used to bound the amount of predictability of returns in tests of the Efficient Market Hypothesis (EMH), as shown by Ross (2005). One of the benefits of this approach over conventional testing methodologies is that, since the computation of the bound in (4e) does not require knowledge of the representative investor's efficient market portfolio, it allows to by-pass Roll's (1977) critique.

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recently, of dynamic option-like strategies and non-linear payoffs, while assuming a frictionless economy.

Table 1
SDF Volatility Bounds

Panel A
(Volatility Bounds Calculations)

$R_{m,t+1}^{max}$	$R_{m,t+1}^{max}$ Period	RRA_V	$b_{V1,min}$	$b_{V2,max}$	σ^2(m)	σ(m) annualized
			1952-2002 Quarterly:			
22.8%	Q1 1975	5.0	-5.00	10.96	0.14	**0.75**
22.8%	Q1 1975	6.4	-6.40	14.04	0.23	**0.96**
			1952-2002 Monthly:			
16.1%	Oct. 1974	5.0	-5.00	7.76	0.05	**0.74**
16.1%	Oct. 1974	6.4	-6.40	9.94	0.07	**0.95**
			1927-2005 Quarterly:			
80.7%	Q2 1933	5.0	-5.00	3.10	0.25	1.00
80.7%	Q2 1933	6.4	-6.40	3.97	0.41	1.28
			1927-2005 Monthly:			
38.2%	Apr. 1933	5.0	-5.00	3.27	0.08	0.95
38.2%	Apr. 1933	6.4	-6.40	4.19	0.12	1.22

Panel B
(Input Factor Variance-Covariance Matrices)
Quarterly Data

$$Var(f)_{1926-2002} = \begin{bmatrix} 1.31 & 0.35 \\ 0.35 & 0.34 \end{bmatrix} \qquad Var(f)_{1952-2002} = \begin{bmatrix} 0.65 & 0.03 \\ 0.03 & 0.01 \end{bmatrix}$$

Monthly Data:

$$Var(f)_{1926-2002} = \begin{bmatrix} 0.31 & 0.01 \\ 0.01 & 0.01 \end{bmatrix} \qquad Var(f)_{1952-2002} = \begin{bmatrix} 0.18 & 0.00 \\ 0.00 & 0.00 \end{bmatrix}$$

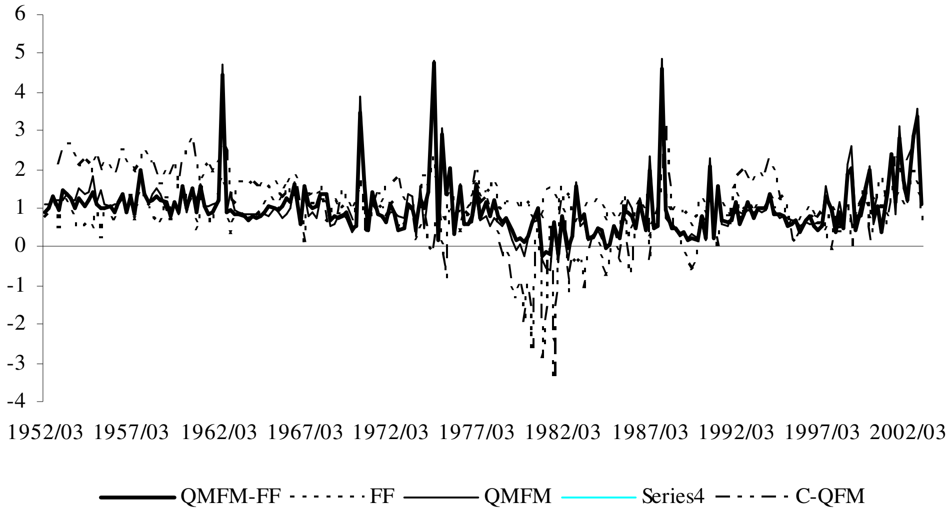
Notes. Panel A of this Table summarizes assumptions and computed values for the SDF volatility bound under different assumptions, corresponding to different sub-sample periods. Panel B reports the variance-covariance matrix estimates used in computing the bounds reported in Panel A. The factors are the market return and its square. All the variables are defined as in the text. The data frequency is either quarterly or monthly.

Table 2
Second Pass Regressions (1952-2002)
30 Industry Portfolios

Model	r_{mt+1}	q_{t+1}	SMB	HML	R^2	Adj. R^2	σ(m)
			Panel A (Beta Pricing Representation)				
LMFM	1.71				3.7	3.7	
	(2.85)						
QMFM	1.90	-0.66			31.2	28.7	
	(3.12)	(-1.91)					
QMFM	2.31	-0.34			13.5	10.5	
1926-2002	(2.30)	(-1.35)					
FFM	2.15		-0.34	-0.66	10.1	3.4	
	(3.64)		(-0.74)	(-1.22)			
QMFM+FFM	2.09	-0.54	-0.08	-0.71	35.6	28.1	
	(3.52)	(-1.70)	(-0.17)	(-1.19)			
			Panel B (Covariance Pricing Representation)				
LMFM	-2.51						39.3
	(-2.83)						
QMFM	-4.08	47.51					131.0
	(-2.86)	(4.27)					
QMFM	-2.78	3.82					55.6
1926-2002	(-3.72)	(2.07)					
FFM	-4.15		3.86	0.20			64.5
	(-3.71)		(2.24)	(0.12)			
QMFM+FFM	-4.78	40.00	2.38	0.12			113.5
	(-3.12)	(3.47)	(1.20)	(0.473)			

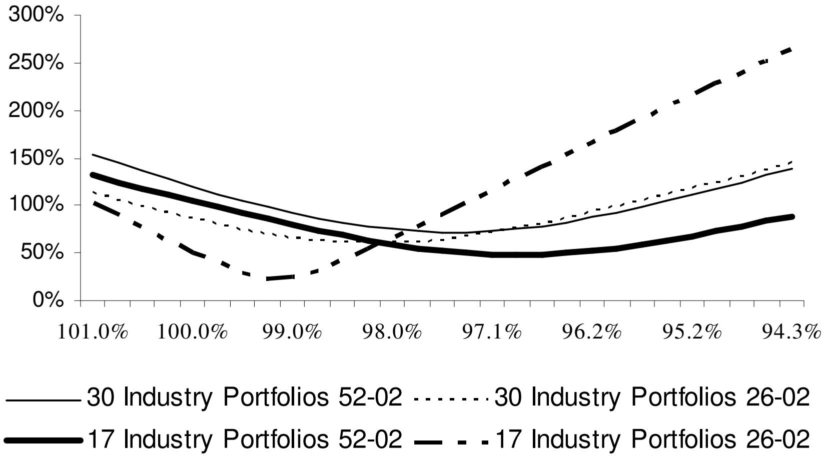
Notes. Panel A of this Table reports 2-step regression estimates of the beta-pricing representation of various factor models for the period 1952-2002. The second pass regressions are estimated without an intercept term. The top row indicates the factors included in each model. For each included factor, I report the risk premia point estimates in percentage and t-statistics in brackets. These are computed using OLS standard errors that account for correlated errors across test portfolios and using Shanken (1992) correction for the fact that the beta coefficients are estimated. The third and second last columns report the coefficient of determination R^2, both unadjusted and adjusted for the degrees of freedom, in percentage. Panel B reports the elements of the *b* vector, the negative of the risk prices, implied by the 2-pass regression estimates (without intercept in the second pass regressions) and, in brackets, associated t-statistics. These are computed using standard errors based on a specification of the *S* matrix that does not allow for serially correlated pricing errors. The last column reports the annualized volatility of the stochastic discount factor in percentage. The market Sharpe ratio is 40.4 percent in 1952-2002 and 35.2 in 1926-2002. All the variables are defined as in the text. The data frequency is quarterly.

Figure 1
Estimated SDF



Notes. This Figure reports the SDF time-series implied by the 2-step regression point estimates of the QMFM, FFM, the QMFM+FFM and the C-QFM. The data is quarterly data for the period 1952-2002.

Figure 2
SDF Mean-Standard Deviation Frontier
(1952-2002)



Notes. This Figure plots the Hansen and Jagannathan (1991) Mean-Standard Deviation frontier of the SDFs that price the quarterly returns on the 17 and 30 industry portfolios for the period 1952-2002. The annualized SDF mean is on the horizontal axis whereas the annualized SDF volatility is on the vertical axis.

Table 3
GMM Estimates with Volatility Bound – Covariance-pricing representation
30 Industry Sorted Portfolios

Model	r_{mt+1}	q_{mt+1}	SMB	HML	TJ_T	TJ_T*	THJ_T	σ(m)
FFM	-2.72		2.00	1.54	21.30	19.84	20.73	50.0
	(0.007)		(0.138)	(0.184)	(0.749)	(0.799)		
	-3.39		2.81	2.54	20.86	20.91	21.65	67.2
	(0.002)		(0.053)	(0.075)	(0.749)	(0.747)		
QMFM	-2.06	17.37			20.27	16.86	21.92	50.0
	(0.052)	(0.069)			(0.872)	(0.913)		
	-2.41	28.55			16.59	15.40	21.22	75.0
	(0.024)	(0.005)			(0.956)	(0.949)		
	-2.61	39.48			14.76	15.04	22.02	100.0
	(0.025)	(0.000)			(0.981)	(0.957)		
QMFM+FFM	-2.20	15.51	1.12	0.87	18.97	16.18	20.00	50.0
	(0.040)	(0.093)	(0.273)	(0.310)	(0.838)	(0.932)		
	-2.55	27.24	1.10	0.97	16.00	14.67	19.33	75.0
	(0.033)	(0.010)	(0.283)	(0.294)	(0.936)	(0.963)		
	-2.73	38.67	0.96	0.84	14.39	14.39	20.43	100.0
	(0.033)	(0.001)	(0.313)	(0.322)	(0.967)	(0.967)		

Notes. This Table reports GMM parameter estimates with positivity and volatility bound on the SDF of various factor models for the period 1952-2002. For each included factor, I report the corresponding b_k (the negative of the risk price) point estimate and its p-value in brackets. Two sets of TJ_T statistics with p-value in brackets are reported. The first are Hansen's (1982) TJ_T statistics. The second sets of TJ_T statistics are calculated using a common weighting matrix for all models and the S matrix of QMFM+FFM with a 100 percent bound on the SDF volatility. In the second last column I report Hansen and Jagannathan (1997) distance, never significant at any conventional level. The last column reports the annualized volatility of the stochastic discount factor in percentage. The market portfolio Sharpe ratio is 40.4 percent. All the variables are defined as in the text.

Table 4
GMM Estimates with Volatility Bound – Beta Pricing Representation
30 Industry Sorted Portfolios

Model	r_{mt+1}	q_{mt+1}	SMB	HML	R^2	Adj. R^2	σ(m)
FFM	1.77		-0.02	-0.90	14.5	4.6	50.0
	(3.04)		(-0.04)	(-1.96)			
	2.26	0.00	-0.09	-1.30	16.3	6.6	67.2
	(3.83)	(-0.02)	(-0.19)	(-2.30)			
QMFM	1.08	-0.23			21.8	16.0	50.0
	(1.86)	(-1.10)					
	1.11	-0.40			31.6	29.2	75.0
	(1.84)	(-1.26)					
	1.04	-0.56			36.0	33.7	100.0
	(1.71)	(-1.71)					
QMFM+FFM	1.16	-0.20	0.01	-0.59	33.0	22.3	50.0
	(2.00)	(-0.97)	(0.03)	(-1.28)			
	1.20	-0.38	0.01	-0.65	40.7	33.9	75.0
	(2.04)	(-1.26)	(0.02)	(-1.14)			
	1.12	-0.55	0.00	-0.62	42.2	35.6	100.0
	(1.88)	(-1.76)	(0.00)	(-1.06)			

Notes. This Table reports the percentage risk premia corresponding, in a beta-pricing representation, to the GMM estimates with positivity and volatility bound on the SDF. For each included factor, I report the corresponding λ_k point estimate and t-statistic in brackets. These are computed using OLS standard errors that account for correlated errors across test portfolios and Shanken's (1992) correction for the fact that the beta coefficients are estimated. The third and second last two columns report the percentage coefficient of determination R^2, both unadjusted and adjusted for the degrees of freedom. The last column reports the SDF volatility.

Table 5
SDF Estimates with Hansen and Hansen and Jagannathan (1997) Weighting Matrix –Augmented Test Asset Payoffs

Weighting Matrix / Restrictions	r_{mt+1}	q_{mt+1}	TJ_T	THJ_T	σ(m)
		17 Industry Portfolios			
LMFM:					
Hansen (1982)	-2.37		17.04		39.3
	(0.000)		(0.316)		
Hansen and Jagannathan (1997)	-2.37			17.91	39.3
	(0.000)			(0.320)	
QMFM:					
Hansen (1982)	-2.92	20.25	13.78		63.5
	(0.000)	(0.220)	(0.541)		
Hansen and Jagannathan (1997)	-2.91	19.89		16.37	62.8
	(0.000)	(0.215)		(0.520)	
	17 Industry Portfolios + Managed Portfolios ($cay_{t-1}r_{it}$)				
LMFM:					
Hansen (1982)	-2.37		54.52		39.3
	(0.000)		(0.007)		
Hansen and Jagannathan (1997)	-2.37			61.37	39.3
	(0.000)			(0.000)	
QMFM:					
Hansen (1982)	-2.21	-3.22	55.72		40.2
	(0.000)	(0.728)	(0.005)		
Hansen and Jagannathan (1997)	-2.21	-5.83		61.00	41.9
	(0.000)	(0.544)		(0.000)	
C-QFM:					
Hansen (1982)					
Hansen and Jagannathan (1997)					
	17 Industry Portfolios + 17 Industry option Payoffs				
LMFM:					
Hansen (1982)	-4.71		151.98		78.2
	(0.000)		(0.000)		
Hansen and Jagannathan (1997)	-4.71			170.31	78.2
	(0.000)			(0.000)	
Hansen and Jagannathan (1997)					
QMFM:					
Hansen (1982)	-15.60	50.28	69.42		266.9
	(0.000)	(0.000)	(0.000)		
Hansen and Jagannathan (1997)	-2.06	-11.25		144.47	48.1
	(0.000)	(0.000)		(0.000)	
Hansen and Jagannathan (1997)	-4.71	0.000		170.31	78.2
$b_2 > 0$	(0.000)	(0.890)		(0.000)	

Notes. This Table reports GMM parameter estimates, under exact pricing of the market factor (see Appendix C for details on how this restriction is imposed), of the unconditional linear and quadratic market factor models. The sample period is 1952-2002. For each included factor, I report the corresponding b_k (the negative of the risk price) point estimate and its p-value in brackets. Two sets of statistics with p-value in brackets are reported. The first are Hansen's (1982) TJ_T statistics. The second sets of statistics are calculated using Hansen and Jagannathan (1997) second moment matrix and are denoted by THJ_T. The last column reports the annualized volatility of the stochastic discount factor in percentage. The market portfolio Sharpe ratio is 40.4 percent. All the variables are defined as in the text.

Table 6
Factor-Conditioning Variables Percentage Correlations
1952-2002

	$r_{m,t+1}$	$r_{m,t}$	$r_{m,t-12}$	$r_{m,t-24}$	cay_t	$r_{TB3M,t}$	$r_{TB1M,t}$	s_t
$r_{m,t+1}$	100.0							
$r_{m,t}$	5.4	100.0						
$r_{m,t-12}$	4.2	-3.1	100.0					
$r_{m,t-24}$	-11.6	-1.5	0.8	100.0				
cay_t	28.6	32.8	-8.4	-3.7	100.0			
$r_{TB3M,t}$	-8.1	-15.8	1.5	-3.9	-1.2	100.0		
$r_{TB1M,t}$	-12.2	-11.6	0.6	-5.2	-2.4	95.7	100.0	
s_t	9.6	-18.2	3.5	2.9	3.1	47.1	19.5	100.0

	$r_{m,t+12}$	$r_{m,t}$	$r_{m,t-12}$	$r_{m,t-24}$	cay_t	$r_{TB3M,t}$	$r_{TB1M,t}$	s_t
$r_{m,t\rightarrow t+12}$	100.0							
$r_{m,t}$	-16.2	100.0						
$r_{m,t-12}$	-0.5	1.2	100.0					
$r_{m,t-24}$	6.4	0.7	0.8	100.0				
cay_t	54.2	28.9	-0.9	-3.7	100.0			
$r_{TB3M,t}$	10.1	-20.1	2.4	-3.0	-7.6	100.0		
$r_{TB1M,t}$	8.4	-15.8	1.5	-4.3	-9.1	95.5	100.0	
s_t	8.6	-19.5	3.7	2.8	2.0	47.1	18.8	100.0

	$r_{m,t+24}$	$r_{m,t}$	$r_{m,t-12}$	$r_{m,t-24}$	cay_t	$r_{TB3M,t}$	$r_{TB1M,t}$	s_t
$r_{m,t\rightarrow t+24}$	100.0							
$r_{m,t}$	-12.7	100.0						
$r_{m,t-12}$	5.6	0.6	100.0					
$r_{m,t-24}$	2.1	-0.1	1.9	100.0				
cay_t	52.3	32.7	2.4	-4.7	100.0			
$r_{TB3M,t}$	16.8	-20.0	2.9	-2.3	-11.4	100.0		
$r_{TB1M,t}$	13.3	-15.7	1.9	-3.7	-13.0	95.5	100.0	
s_t	16.1	-19.1	3.9	3.5	1.2	46.8	18.4	100.0

Notes. This Table reports percentage estimates of the correlation between factors and various predictive variables. The sample period is 1952-2002. The symbol $r_{m,t\rightarrow t+k}$ denotes the return from t to t+k (a multi-period return), $r_{TB3M,t}$ and $r_{TB1M,t}$ are the yields on the 3 and 1 month T-Bill, respectively, and s_t is the spread between the former and the latter. All the other variables are defined as in the text. Slight differences in the reported correlations across the three panels are due to the fact that the sample period changes depending on the amount of observations available.

Table 7
GMM Estimates – Covariance-pricing representation
17 Industry Sorted Portfolios + Managed Portfolios

Model	r_{mt+1}	$r_{mt+1}cay_t$	$r_{mt+1}\,s_t$	Δy_{t+1}	$\Delta y_{t+1}\,cay_t$	$\Delta y_{t+1}\,s_t$	SMB_{t+1}	$SMB_{t+1}cay_t$	$SMB_{t+1}\,s_t$	HML_{t+1}	$HML_{t+1}cay_t$	$HML_{t+1}s_t$	THJ_T	σ(m)
C-FFM	-2.0	-14.8	-528.3	27.0	-884.9	-14368.8	0.2	-33.2	597.7	0.6	33.9	-216.4	173.0**	46.0
m > 0	(0.101)	(0.879)	(0.529)	(0.819)	(0.862)	(0.750)	(0.925)	(0.812)	(0.693)	(0.801)	(0.731)	(0.814)		
	6.6	0.0	0.0	-0.0	0.0	0.0	0.8	0.0	0.0	-1.9	0.01	-0.0		

Model	r_{mt+1}	$r_{mt+1}cay_t$	$r_{mt+1}s_t$	q_{mt+1}	$q_{mt+1}cay_t$	$q_{mt+1}s_t$	Δy_{t+1}	$\Delta y_{t+1}cay_t$	$\Delta y_{t+1}s_t$	q_{yt+1}	$q_{yt+1}cay_t$	$q_{yt+1}s_t$	THJ_T	σ(m)
C-QMFM	-5.4	-143.2	109.4	-2.9	1212.5	-5643.2	-92.7	-975.9	-44011.8	165.5	-465.6	-6272.8	103.8	224.0
	(0.002)	(0.150)	(0.893)	(0.870)	(0.205)	(0.354)	(0.409)	(0.836)	(0.311)	(0.000)	(0.580)	(0.157)		
	6.6	0.0	0.0	-1.6	-0.0	0.0	0.1	0.0	0.0	-2.6	-0.0	0.0		
C-QMFM	-3.5	-132.3	260.8	-3.5	1111.7	-3030.3	-17.2	-201.6	-41265.1	114.6	-303.8	-1560.0	109.9*	154.0
m > 0	(0.037)	(0.183)	(0.749)	(0.838)	(0.244)	(0.619)	(0.855)	(0.963)	(0.201)	(0.000)	(0.718)	(0.709)		
	3.1	0.0	0.0	-1.0	-0.0	0.0	0.0	0.0	0.0	-1.9	-0.0	0.0		
C-QMFM	-4.3	-135.7		-12.9	715.9					140.8			116.4*	185.8
m > 0	(0.000)	(0.072)		(0.274)	(0.282)					(0.000)				
	6.6	0.0		-1.0	0.0					-2.3				

Notes. This Table reports GMM parameter estimates of various factor models for the period 1952-2002, under exact pricing of the stock market factor (see Appendix C for details on how this restriction is imposed). For each included factor, I report the corresponding b_k (the negative of the risk price) point estimate and its p-value in brackets. I also report Hansen and Jagannathan (1997) distance. I denote by ** and * significance at the 1 and 5 percent level. The last column reports the annualized volatility of the stochastic discount factor in percentage. The market portfolio Sharpe ratio is 40.4 percent. The conditioning variable cay is scaled by a factor of 100. All the variables are defined as in the text.

Table 8
Second Pass Regressions with Intercept (1952-2002)
Beta-Pricing Representation
Industry Portfolios

Model	α	r_{mt+1}	q_{mt+1}	SMB	HML	R^2	Adj. R^2	σ(m)
LMFM	1.44	0.44				3.8	0.4	10.3
	(3.36)	(0.73)						
		(0.75)						
QMFM	1.03	0.91	-0.51			36.1	31.3	91.2
	(2.87)	(1.51)	(-1.59)					
		(1.56)	(-2.23)					
FFM	2.27	-0.20		0.16	-0.79	24.8	16.1	31.6
	(3.25)	(-0.34)		(0.34)	(-1.43)			
		(-0.34)		(0.38)	(-1.73)			
QMFM+FFM	1.71	0.32	-0.49	0.25	-0.77	46.9	38.5	85.5
	(2.82)	(0.54)	(-1.60)	(0.50)	(-1.34)			
		(0.56)	(-2.23)	(0.59)	(-1.65)			

Notes. This Table reports the beta-pricing representation of 2-step regression estimates of various factor models for the period 1952-2002. The second pass regressions are estimated with an intercept term, denoted by the symbol α. The top row indicates the factors included in each model. For each included factor, I report the risk premia point estimates in percentage and two t-statistics in brackets. The first set of t-statistics are computed using OLS standard errors that account for correlated errors across test portfolios and using Shanken (1992) correction for the fact that the beta coefficients are estimated. The second set of t-statistics use GLS standard errors. The last three columns report the coefficient of determination R^2 (both unadjusted and adjusted for the degrees of freedom) and the annualized volatility of the stochastic discount factor in percentage. The market Sharpe ratio is 40.4 percent. All the variables are defined as in the text. The data frequency is quarterly.

Appendix A: Bounding the Curvature of *V*

Consider the MU corresponding to a third order Taylor expansion of the utility transformation $V(W_{t+1})$,

$$m_{Vt+1}(R_{t+1}) \cong 1 + b_{V1} R_{t+1} + b_{V2} R_{t+1}^2 \tag{A1}$$

Here, $R_{t+1} \equiv \frac{W_{t+1}}{W_t}$, $b_{V1} \equiv \frac{1}{2} V''(W_t) W_t$, $b_{V2} \equiv \frac{1}{6} V'''(W_t) W_t^2$, and W_t is an initial wealth level around which $V_t(W_{t+1})$ is expanded in a Taylor series. Normalizing this level to one, $V_t(W_{t+1})$ is standardized in such a way that $V(W_t) = V(1) = 0$ and $V'(W_t) = V'(1) = 1$. This standardization is legitimate since utility functions are unique only up to a linear transformation. For m_{Vt+1} to be decreasing in R_{t+1}, its first derivative should satisfy the following condition:

$$\frac{\partial m_{Vt+1}}{\partial R_{t+1}} = b_{V1} + 2 b_{V2} R_{t+1} < 0 \tag{A2}$$

Treating V as a standardized utility function, note that $RRA_V \cong -\kappa \frac{\partial m_{Vt+1}}{\partial R_{t+1}}$, where $\kappa = \frac{1 + R_{t+1}}{m_{Vt+1}}$. Using this fact, imposing (A2), and letting $RRA_V^{\max}$ denote an upper bound on relative risk aversion, we can write

$$0 > b_{V1} + 2b_{V2} R_{t+1} \geq -\frac{RRA_V^{\max}}{\kappa} \qquad \kappa > 0 \tag{A3}$$

The following is a necessary condition for the first inequality in (A3) to be satisfied over all the possible (positive) values of R_{t+1}:

$$b_{V2} < -\frac{1}{2}\frac{b_{V1}}{R_{t+1}^{\max}} \qquad R_{t+1}^{\max} = Max(R_{t+1} \mid prob(R_{t+1}) > 0) \tag{A4}$$

In turn, the second inequality in (A3) implies $b_{V1} \geq -\frac{RRA_V^{\max}}{\kappa} - 2b_{V2} R_{t+1}$. A necessary and sufficient condition for this condition to hold is that $b_{V1} \geq -\frac{RRA_V^{\max}}{\kappa} + Max(-2b_{V2}R_{t+1})$ while a necessary condition for V to display non increasing absolute risk aversion (NIARA) is $b_{V2} \geq 0$. Together, these conditions imply $b_{V1} \geq -\frac{RRA_V^{\max}}{\kappa} - 2b_{V2}R_{t+1}^{\min}$, where $R_{V,t+1}^{\min} = Min(R_{t+1})$ denotes the minimum value of the range over which the representative investor's market return probability density function is defined. Making the reasonable assumption that the minimum of the market return is a negative number, a necessary condition for this inequality to hold is that:

$$b_{V1} \geq -\frac{RRA_V^{\max}}{\kappa} \tag{A5}$$

Thus, combining (A4) and (A5),

$$b_{V2} \leq \frac{1}{2}\frac{RRA_V^{\max}}{\kappa R_{t+1}^{\max}} \tag{A6}$$

Finally, (A5) and (A6) must hold for any value of $\kappa = \frac{1+R_{t+1}}{m_{Vt+1}}$, including $\frac{1+R_{t+1}^{\max}}{m_{Vt+1}(R_{t+1}^{\max})}$.

For realistic return processes, it is typically[24] the case that $R_{t+1}^{\max} \geq 0$. Moreover, since V is concave in wealth, $m_{Vt+1}(R_{t+1}^{\max}) = V'(W_{t+1}^{\max}) \leq V'(W_t)$. Furthermore, since V is a standardized utility function, $V'(W_t) = 1$ and therefore $m_{Vt+1}(R_{t+1}^{\max}) \leq 1$. Thus, $\frac{1+R_{t+1}^{\max}}{m_{Vt+1}(R_{t+1}^{\max})} \geq 1$ and[25] the following are necessary conditions for (A5) and (A6), respectively, to hold:

$$b_{V1} \geq -RRA_V^{\max} \equiv b_{V2}^{\min} \qquad b_{V2} \leq \frac{1}{2}\frac{RRA_V^{\max}}{R_{t+1}^{\max}} \equiv b_{V2}^{\max} \tag{A7}$$

[24] This is actually a necessary condition for no-arbitrage. In fact, if the maximum possible value of the return on an asset was negative, selling the asset would allow a risk-less profit (i.e., a positive payoff with non-zero probability at no cost).

[25] This amounts to imposing a further restriction on the curvature of *V* and thus on the volatility of MU.

Appendix B: Maximal Sharpe Ratios from Option Portfolios

As shown in Table 2, the unconditional volatility of m_{t+1} for the QMFM is more than three times as large as the market SR. Even though the SR is not an exhaustive criterion to rank risky alternatives in a non mean-variance world, such a difference is hard to justify. Because of the very high absolute value of the correlation between the SDF and the market excess return (about -90% in the 1952-2002 period), a strategy perfectly correlated to the SDF could have such a higher SR than the market portfolio only if the average investors particularly disliked exposure to very unlikely states of the world. To see this, let us denote by x^*_{t+1} the excess-returns on the portfolios perfectly correlated with the SDF. By Hansen and Jagannathan's (1991) bound, these are the portfolios that offer the maximal ex-ante SR, i.e. $\frac{E_t(x^*_{i,t+1})}{\sigma_t(x^*_{i,t+1})} = -(1+R_{f,t})\sigma_t(m_{t+1}) = SR^*_t$. As in Goetzmann, Ingersoll, Spiegel and Welch (2004), it is straightforward to show that:

$$x^*_{i,t+1} = E_t(x^*_{i,t+1}) - \frac{E_t(x^*_{i,t+1})}{SR^*_t}(m_{t+1} - 1) \qquad \text{(B1)}$$

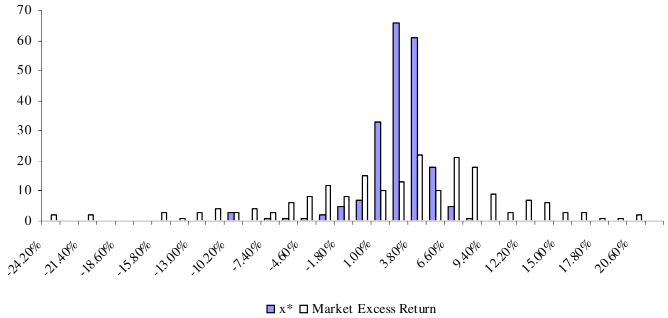
In the context of the 3M-CAPM, m_{t+1} is the SDF of the representative investor. Using the QMFM point estimates of the elements of b reported in Panel B of Table 2, I construct a time series of sample realizations of m_{t+1} (plotted in Panel A of Figure 4). Using (B1) and setting $E_t(x^*_{i,t+1}) = E(x^*_{i,t+1}) = E_T(r_{m,t+1})$ and $SR^*_t = SR^* \cong \sigma(m_{t+1})$, I then construct the time series $x^*_{i,t+1}$ of the excess returns on portfolio i, i.e. one of the maximal SR portfolios. As can be seen from Figure 3, the maximal SR portfolio excess return histogram has a much less pronounced right hand tail, but moderately positive realizations are more frequent than in the histogram of the market excess return. In other words, this maximal SR portfolio underperforms the market portfolio on the upside but it offer better protection on the downside.

I use this observation to place an empirical bound on the SR achievable by $x^*_{i,t+1}$ by constructing a replicating portfolio, given the market SR and no-arbitrage prices of call and put options on the market portfolio. More specifically, I ask whether we can achieve a SR as large as SR^* by means of selling the upside of the market portfolio (this is all we have to sell as the downside of the market portfolio is worse than the downside of the maximal SR portfolio). To be able to compare the replicated and the replicating portfolio, I 'fix' one of the moments that appear in the SR. In particular, I make sure that the replicating portfolio displays at least the same volatility as the maximal SR portfolio. In other words, starting from a strategy that resembles the stock market index, I attempt to increase its SR ratio by seeking a higher expected excess return rather than by lowering volatility. Thus, I search for a strategy that replicates the maximal SR portfolio as closely as possible, that is exposed to no less volatility, and that offers a SR as high as possible. I also make sure that the histogram of the replicating strategy is never above but close to the histogram of $x^*_{i,t+1}$ in correspondence of any non negative bin and never below in correspondence of negative bins. The latter condition ensures that, while the replicating strategy is dominated by $x^*_{i,t+1}$, its SR is not kept low by lack of exposure to states of the world that investors particularly dislike. Panel B plots the histogram of a strategy that satisfies these conditions under the assumption that implied volatility is 20 percent per annum. The strategy is 100 percent long the market portfolio, 80 percent short calls and 42 percent long puts on the market portfolio. The annual SR of this strategy is 53.25 percent. Panel C plots the histogram of a strategy that satisfies these conditions assuming an implied volatility of 25 percent per annum. The strategy is 100 percent long the market portfolio, 80 percent short calls and 46 percent long puts on the market portfolio. The annual SR of this strategy is 74.71 percent. Options have a strike price equal to 99 percent of the market portfolio forward price, expire in 3 months and are valued using the Black and Scholes (1973) model without dividends.

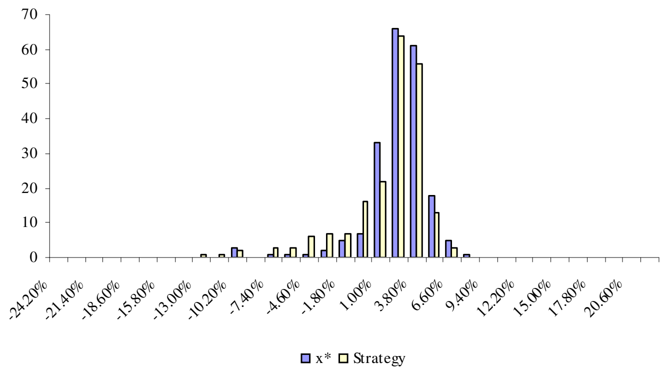
The SR of these strategies is considerably lower than SR^*, yet their payoffs are very close to the payoff of the maximal SR portfolio. Thus, the small differences between the payoffs of these strategies and the maximal SR portfolio payoff must account for the difference between their SR and SR^*. This appears unlikely. A SR of the replicating strategies closer to SR^* would require a higher implied volatility, about 35 percent. This level of implied volatility appears unrealistic over a prolonged period of time, unless we allow for unusually high volatility risk premia. This suggests that the excess return process on the maximal SR portfolio itself is unfeasible and, as a result, the volatility of the SDF should be lower. Interestingly, as shown in the comprehensive study of Santa Clara and Saretto (2004), even though it is possible to construct option-based strategies that offer a very high SR, the latter is considerably lower after taking transaction costs and no-short sale constraints into account. This represents further, indirect evidence that the truly attainable maximal SR is somewhat lower than SR^*, even taking complex option-based strategies into account.

Figure 3
Market and Maximal SR Portfolios Histograms

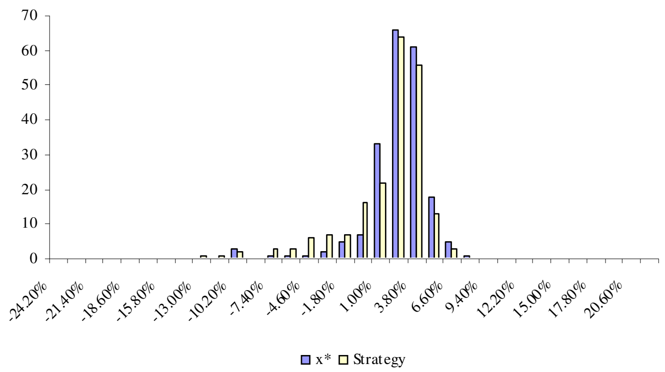
Panel A



Panel B



Panel C



Notes. Panel A of this Figure plots the histograms of the market portfolio and maximal SR portfolio excess returns. Panel B plots the histograms of the maximal SR portfolio excess returns and of the excess returns on a strategy that is 100 percent long the market portfolio and 80 percent short calls and 42 percent long put on the market portfolio. Implied volatility is assumed to be 20 percent. Panel C plots the histograms of the maximal SR portfolio excess returns and of the excess returns on a strategy that is 100 percent long the market portfolio and 80 percent short calls and 46 percent long put on the market portfolio. Implied volatility is assumed to be 25 percent. Options have a strike price equal to 99 percent of the market portfolio forward price, expire in 3 months and are valued using the Black and Scholes (1973) model without dividends.

Appendix C: OLS vs. GLS

Estimating the beta-pricing representation of a factor model, as shown by Jagannathan and Wang (2002), is equivalent in terms of consistency and asymptotic efficiency to directly estimating its SDF-representation. Under appropriate conditions, the minimization of the pricing error metric in (5) yields the same estimates as a classic 2-pass regression procedure. The parameters of m_{t+1} that solve (5) can be efficiently estimated by GMM. This entails using the inverse of the following matrix as the weighting matrix for the moment conditions:

$$\begin{aligned} S &\equiv \sum_{j=-\infty}^{\infty} E\left[u_t(b_t)u_{t-j}(b_t)'\right] \\ u_{t+1}(b_t) &= m_{t+1}(b_t)x_{t+1} - p_t \end{aligned} \quad \text{(C1)}$$

As shown by Hansen (1982), S^{-1} is the efficient choice of the weighting matrix and, under relatively mild regularity conditions, it can be estimated consistently by iterated GMM. Other choices for the weighting matrix are, however, admissible, e.g. $W = I$ or, as advocated by Hansen and Jagannathan (1997), $W = E(x_{t+1}x'_{t+1})^{-1}$. Using $W = I$ yields OLS estimates, setting W equal to a 1-step estimate of the inverse of S yields a feasible GLS, and $W = E(x_{t+1}x'_{t+1})^{-1}$ is an instance of weighted least squares. Adding the constraint $E_T(m_{t+1}f_{t+1}) = E_T\left[p_t(f_{t+1})\right]$ and thus imposing exact factor pricing also yields GLS second pass regression estimates. This is because GLS assigns the largest weights to the moments estimated with most precision. Any volatility constraint in excess of the (discounted) SR of the exactly priced factor with the largest SR is slack whereas, for any SR below the latter, (5) has no solution. Adding the constraint $E_T(m_{t+1}x_{EW\,t+1}) = E_T\left[p_t(x_{EW\,t+1})\right]$ instead forces the model to assign zero pricing error to the equally weighted average payoff $x_{EW\,t+1}$ and thus yields OLS second pass regression estimates. Any volatility constraint in excess of the (discounted) SR of $x_{EW\,t+1}$ is slack whereas, for any SR below the latter, (5) has no solution. When working with excess returns, this constraint reduces to $E_T(m_{t+1}r_{EW\,t+1}) = E_T\left[p_t(r_{EW\,t+1})\right] = 0$, where $r_{EW\,t+1}$ is the excess return on the equally weighted portfolio of the test assets.

By construction, all the cross-sectional variation in historical average excess returns captured by a second pass regression without intercept is explained by variation in factor loadings. This is equivalent to estimating without SDF volatility upper bound. On the other hand, when estimating the second pass regression with an intercept, the latter explains a portion of the variation in average returns across stocks. Thus, the factor loadings and therefore the estimated asset pricing model are left with the task of explaining a lower portion of this variation. By (2), this requires a less volatile SDF and thus it is equivalent to estimating under a volatility upper bound.

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