MM203: Problem solving tutorial sheets

Note that the questions given below are a selection of questions, some taken from the following books:

Engineering Mechanics: Dynamics, Meriam & Kraige, Wiley

and

An Introduction to Mathematics for Engineers: Mechanics, Lee, Hodder Education

and do not fully represent either the level of, or material covered in, MM203 exams.

Problem solving is a skill and requires practise over a significant period of time. Attendance at tutorials is very important.

Tutorial Sheet 1

QUESTION 1

Calculate the constant acceleration a which the catapult of a stationary aircraft carrier must provide to produce a launch velocity of 360 km/h in a distance of 100 m.

Answer: $a = 50$ m/s^2

QUESTION 2

The motion of a golf ball as it is putted in a straight line towards a hole is to be modelled. From video images, the distance of the ball from the golfer, s, is measured and the results are found to approximately satisfy the following equation

$$s = -\tfrac{3}{2}t^2 + 12t \quad 0 \le t \le 4$$

The hole is 24 m from the golfer.

i. Explain the restriction $0 \le t \le 4$.
ii. Find the velocity of the ball in terms of t.
iii. With what speed does the ball enter the hole?
iv. Find the acceleration of the ball in terms of t.

Answers: ii) $-3t+12$ m/s, iii) 0 m/s, iv) -3 m/s^2

QUESTION 3

A ball is thrown upwards with an initial speed of 10 m/s from a height of 5 m above the ground. It has a constant acceleration vertically downwards of 10 m/s^2 due to gravity.

i. Find the maximum height of the ball above the ground and the time it takes to reach it.
ii. When will the ball hit the ground?

At the instant the ball is thrown a second ball is thrown vertically upwards from ground level with an initial speed of 12.5 m/s.

iii. When will the two balls pass each other?

Answers: i) 10 m, 1 s, ii) 2.414214 s after ball thrown, iii) 2 s after balls thrown

QUESTION 4

A rocket car is launched horizontally along a straight road with an unknown initial velocity and accelerates with an unknown constant acceleration. The velocity is measured at 100 m/s after 10 seconds and at that same instant, the distance travelled is 200 m. Find the acceleration of the car.

Answer: 16 m/s^2

QUESTION 5

If the velocity of a device is given by

$$v = 10 - t^2$$

then find the distance travelled by it between $t = 0$ and $t = 10$.

Answer: −233⅓ m

QUESTION 6

In an archery test, the acceleration of the arrow decreases linearly with distance s from its initial value of 4800 m/s^2 upon release to zero after a horizontal travel of 600 mm (at which point it leaves the bow). Calculate the maximum velocity v of the arrow.

Hints:
Is the acceleration of the arrow constant? If it's constant then you can use the "suvat" equations.
If not, what is it dependent on?
Write an equation for the acceleration.
How do you find the velocity from the acceleration?

Answer: $v = 53.7$m/s

Tutorial Sheet 2

QUESTION 1

Question 12.2 from Lee.

Answer: 3.3 rad/s

QUESTION 2

Question 12.8 from Lee.

QUESTION 3

Question 12.14 from Lee.

Answers: i) 2π rad/s^2, ii) 4 revs

QUESTION 4

A test car starts from rest on a horizontal circular track of 100 m radius and increases its speed at a uniform rate to reach 72 km/h in 10 seconds. Determine the magnitude of the total acceleration of the car 5 seconds after the start.

Answer: $\sqrt{5}$ m/s^2

QUESTION 5

The speed of the car in Figure 1 increases uniformly with time from 72 km/h at A to 144 km/h at B during 10 seconds. The radius of curvature of the hump at A is 40 m. If the magnitude of the total acceleration of the car's mass centre is the same at B as at A, compute the radius of curvature ρ_B of the dip in the road at B. The mass centre of the car is 0.6 m from the road but, for the sake of simplicity, you can assume that it is at road level.

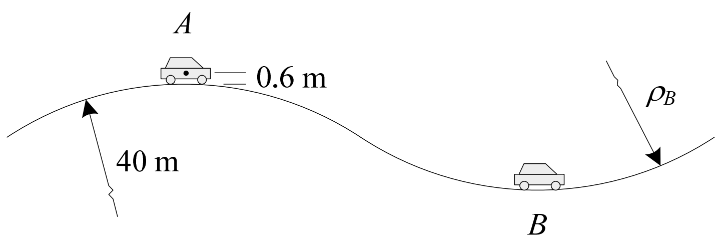

Figure 1: Schematic of car travelling along road.

Answer: ρ_B = 160 m

QUESTION 6

The position vector of a machine component is given by

$$\mathbf{r} = 3t\mathbf{i} - t^2\mathbf{j}$$

i. What is the speed of the component at t = 2 s?
ii. Draw the velocity vector for the component at t = 2 s.
iii. Draw the acceleration vector at t = 2 s.
iv. What is the normal component of the acceleration at t = 2 s?
v. What is the radius of curvature of the motion of the component at t = 2 s?

Answer: i) 5 m/s, iv) 1.6 m/s^2, v) 15.625 m

Tutorial Sheet 3

QUESTION 1

In the design of a timing mechanism shown in Figure 2, the motion of the pin A in the fixed circular slot is controlled by the guide B, which is being elevated with a constant upward velocity, v_0 = 2 m/s for an interval of its motion. Calculate both the normal and tangential components of acceleration of pin A as it passes the position for which $\theta = 30°$.

Hint:

What do you know about the acceleration of pin A? You know that it's moving in a circular path. You also know that it is being moved by the guide. Put this information down on paper using a vector diagram.

Do you need to know the velocity of the pin A? If so, how will you find it? What do you know about its velocity – i.e. about its direction and its magnitude?

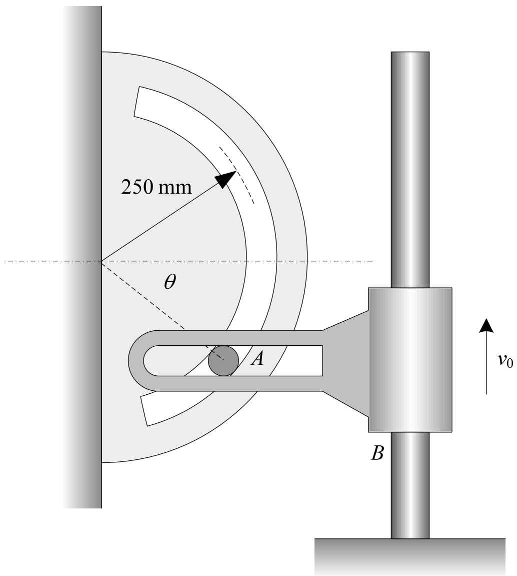

Figure 2: Timing mechanism.

Answer: $a_n = 21.3$ m/s^2, $a_t = -12.32$ m/s^2

QUESTION 2

The x and y positions of a machine part P are controlled by sliders. The x-position in metres over the interval in question is $x = 2t-3$ and the equivalent y position is $y = t^2$ where t is the time in seconds. The angle and distance to part P are measured from the point (0, 0) as shown in Figure 3. Calculate dr/dt in m/s and $d\theta/dt$ in rad/s at time $t = 1$.

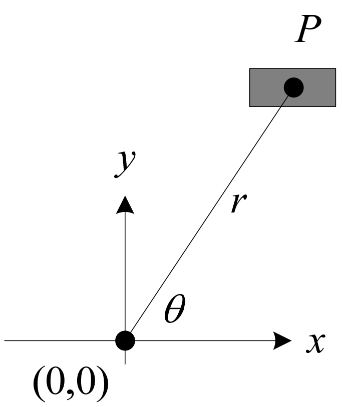

Figure 3: Position of machine part P.

Answer: 0 m/s, 2 rad/s

QUESTION 3

A car P travels along a straight road with a constant speed v = 72 km/h. At the instant when the angle θ = 60°, determine the values of dr/dt in m/s and $d\theta/dt$ in rad/s.

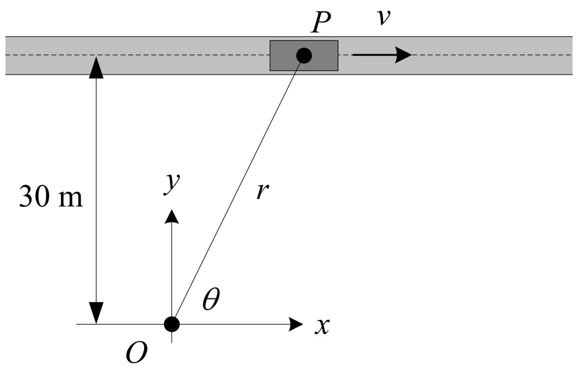

Figure 4: Car travelling along road with coordinate system shown.

Answers: 10 m/s, −0.5 rad/s

QUESTION 4

Returning to the problem in Question 1:

i. Given that the +ve y-direction is vertically upwards, that the acceleration due to gravity is 10 m/s^2, and that the mass of part P is 10 kg, what force are the sliders applying to P at t = 1 s.
ii. Determine d^2r/dt^2 and $d^2\theta/dt^2$ at t = 1 s.

Answers: i) 120 N upwards, ii) 7.071 m/s^2, 1 rad/s^2

QUESTION 5

Car A shown in Figure 5 is travelling at a constant speed of 72 km/h as it rounds a circular curve of 500 m radius. At the instant shown it is at θ = 45°. Car B is passing the centre of the circle at the same instant at a constant speed of 90 km/h. Car A is located relative to B using polar coordinates with the pole moving with B. For this instant, determine $v_{A/B}$ and the values of $d\theta/dt$ and dr/dt as measured by an observer in car B.

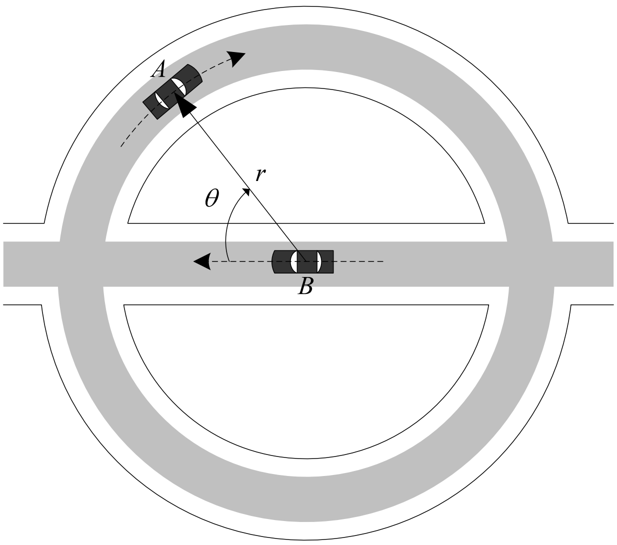

Figure 5: Two cars moving as indicated.

Answers: $v_{A/B}$ = 41.62 m/s, $d\theta/dt$ = 0.075 rad/s, dr/dt = −17.68m/s.

Tutorial Sheet 4

QUESTION 1

Determine the angular velocity of link *BC* for the instant indicated in Figure 6. In case (a), the centre *O* of the disc is a fixed pivot, while in case (b), the disc rolls without slipping on the horizontal surface. In both cases, the disc has clockwise angular velocity ω. Neglect the small distance of pin *A* from the edge of the disc.

Answers: (a) ω CCW, (b) 2ω CCW

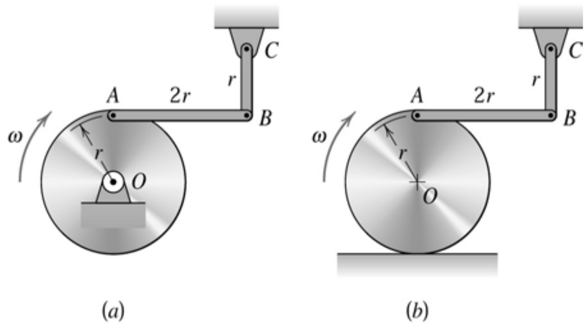

Figure 6: Simple mechanisms.

QUESTION 2

Horizontal motion of the piston rod of the hydraulic cylinder in Figure 7 controls the rotation of link *OB* about *O*. For the instant represented, v_A = 2m/s and *OB* is horizontal. Determine the angular velocity ω of *OB* for this instant.

Answer: ω = 20 rad/s CCW

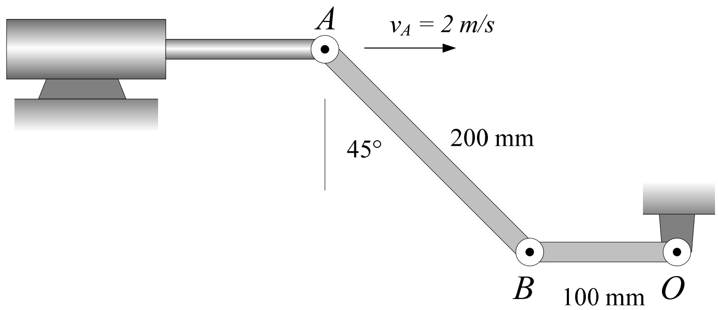

Figure 7: Mechanism.

QUESTION 2

Horizontal motion of the piston rod of the hydraulic cylinder in Figure 8 controls the rotation of link *OB* about *O*. The velocity of *A* is a constant v_A = 2m/s. *OB* is horizontal and *AB* is vertical at the instant shown. Determine the angular velocity and angular acceleration of *OB* for this instant.

Answer: 0 rad/s CCW, 20 rad/s^2 CW

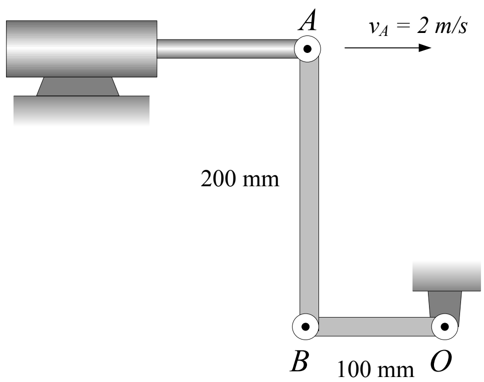

Figure 8: Mechanism.

QUESTION 3

The centre O of the disc in Figure 9 has the velocity and acceleration shown in the figure. If the disc rolls without slipping on the horizontal surface, determine the velocity of A and the acceleration of B for the instant represented.

Hint: first calculate the angular velocity and angular acceleration of the disc.

Answer: $v_A = 5.12\mathbf{i} + 2.12\mathbf{j}$ m/s, $a_B = -16.25\mathbf{i} + 2.5\mathbf{j}$ m/s^2

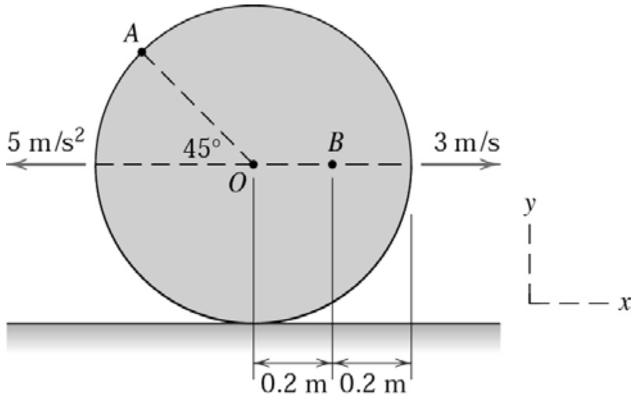

Figure 9: Rolling disc.

QUESTION 4

Question 2.2 from Lee.

QUESTION 5

Question 2.7 from Lee. Assume $g = 10$ m/s^2.

Answer: i) 7.76 N, ii) 8 N

Tutorial Sheet 5

QUESTION 1

Question 2.9 from Lee.

Answers: i) 13 N, ii) 90 m, iii) 13 N

QUESTION 2

Question 5.5 from Lee.

Answers: ii) 4104 N, iii) 9193 N

QUESTION 3

Parts i) and ii) of Question 5.10 from Lee. Assume $g = 10$ m/s^2.

Answers: i) 6.43 m/s^2, ii) 6.94 m/s, iii)

QUESTION 4

A man pulls himself up the 15° incline by the method shown in Figure 10. If the combined mass of the man and the cart is 100 kg, determine the acceleration of the cart if the man exerts a pull of 250 N on the rope. Neglect all friction and the mass of the rope, pulleys, and wheels.

Answer: $a = 4.96$ m/s^2

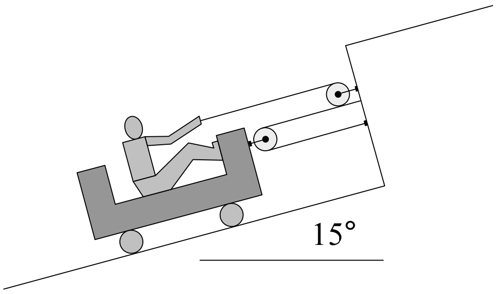

Figure 10: Man in cart with pulley mechanism shown.

QUESTION 5

Determine the proper bank angle θ (see Figure 11) for the airplane flying at 720 km/h and making a turn of 5 km radius. Note that the lift force exerted by the air is normal to the supporting wing surface. The proper bank angle is the one for which the aircraft follows the required circular path and does not "slip" sideways. Assume $g = 10$ m/s^2.

Answer: $\theta = 38.66°$

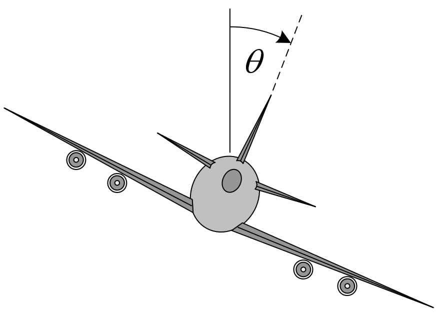

Figure 11: Aircraft with bank angle indicated.

Tutorial Sheet 6

QUESTION 1

Question 9.3 from Lee.

Answers: i) 64 J, iii) 64 N·m, iv) 400 N, v) $\sqrt{8000}$ m/s

QUESTION 2

The mobile wrecking crane in Figure 12 is moving with a constant speed of 3 km/h when it is suddenly brought to a stop. Compute the maximum angle θ through which the cable of the wrecking ball swings.

Answer: $\theta = 6.23°$

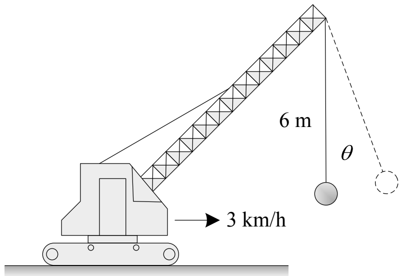

Figure 12: Mobile wrecking crane with angle of swing indicated.

QUESTION 3

In the design of a conveyor-belt system, small metal blocks are discharged with a velocity of 0.4 m/s onto a ramp by the upper conveyor belt shown in Figure 13. If the kinetic coefficient of friction between the blocks and the ramp is 0.30, calculate the angle θ which the ramp must make with the horizontal so that the blocks will transfer without slipping to the lower conveyor belt moving at the speed of 0.14 m/s.

Answer: $\theta = 16.62°$

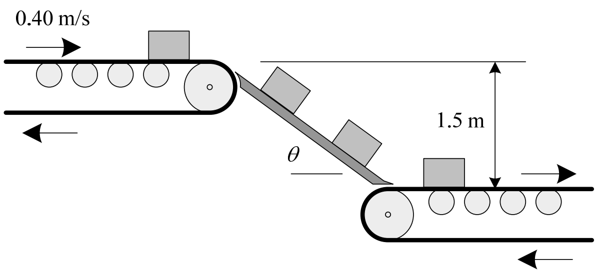

Figure 13: Conveyor-belt system.

QUESTION 4

The 0.60 kg collar in Figure 14 slides on the curved rod in the vertical plane with negligible friction under the action of the constant force F in the cord guided by the two pulleys at D. If the collar is released from rest at A, determine the force F which will result in the collar striking the stop at B with a velocity of 4 m/s. HINT: There is an easy way of determining the work done by the force – i.e. of determining the distance over which it acts.

Answer: $F = 13.21$ N

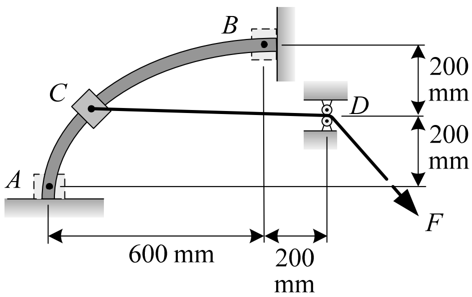

Figure 14: Collar and rod with cord and pulleys shown.

Tutorial Sheet 7

QUESTION 1

Two magnets are released at the same instant with zero velocity on a frictionless surface. Magnet *A* has a mass of 10 mg while magnet *B* has a mass of 2 mg. Due to an attraction force between them (which varies with the distance between them) they begin to move towards each other with zero rotation. At a certain instant mass *A* is moving with a velocity v_A = 2 m/s. At that instant, what is the velocity of magnet *B*?

Answer: v_B = 10 m/s

QUESTION 2

Parts i) to iii) of Question 10.1 from Lee.

Answers: i) 11880 N·s , ii) 99 kN, iii)

QUESTION 3

An old 2000 kg gun (see Figure 15) fires a 10 kg shell with an initial velocity of 600 m/s at an angle of 30° to the horizontal. The gun rests on a horizontal surface and is free to move horizontally. Assuming that the barrel of the gun is rigidly attached to the frame (i.e. that there is no recoil mechanism) and that the shell leaves the barrel 6 ms after firing, determine the recoil velocity of the gun (i.e. it's horizontal velocity immediately after firing) and the increase in the reaction force exerted by the ground on the gun resulting from the firing.

Answer: v_G = 2.60 m/s backwards, *R* = 500 kN

Note that due to the very short time over which the impulsive force is applied to the projectile, non-impulsive forces such as its weight can be ignored. Why is this?

Figure 15: Field cannon.

QUESTION 4

Figure 16 shows two views of a rotating mechanism. The assembly starts from rest and reaches an angular speed of 150 rev/min under the action of a 20 N force *T* applied to the string (around the 100 mm radius pulley) for *t* seconds. Neglect friction and the all masses except for the four equal 3 kg spheres, which may be treated as particles. Determine the value of *t*.

Answer: *t* = 15.08 s

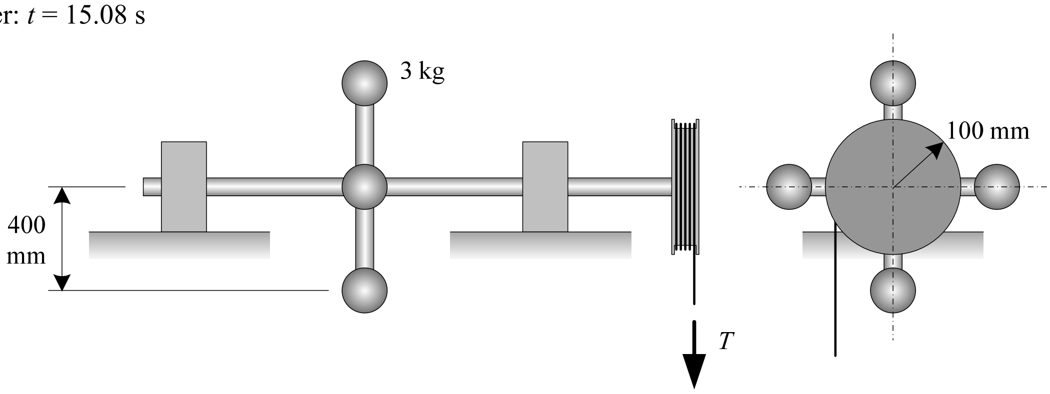

Figure 16: Two views of rotating mechanism.

Tutorial Sheet 8

QUESTION 1

The pendulum shown in Figure 17 consists of two 5 kg concentrated masses positioned as shown on a light rigid bar. The pendulum is swinging through the vertical position with a clockwise angular velocity $\omega = 4$ rad/s when a 50 g bullet travelling with a velocity $v = 500\sqrt{2}$ m/s in the direction shown strikes the lower mass and becomes embedded in it. Calculate the angular velocity ω' which the pendulum has immediately after impact and find the maximum height, h, the lower mass reaches above it's lowest point after the collision. Assume $g = 10$ m/s^2.

Answer: $\omega' = 1.984$ rad/s (CW), $h = 1.964$ m

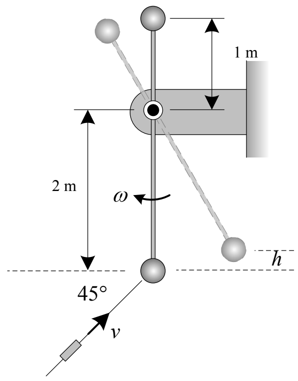

Figure 17: Pendulum and bullet.

QUESTION 2

The flatbed truck in Figure 18 is travelling at the constant speed of 60 km/h up the constant 15-in-100 gradient when the 100 kg crate which it carries is given a shove which imparts to it an initial relative velocity of $dx/dt = 3$ m/s toward the rear of the truck. If the crate slides a distance $x = 2$ m (measured on the truck bed as shown) before coming to rest on the bed, compute the coefficient of kinetic friction μ_k between the crate and the truck bed.

Answer: $\mu_k = 0.382$

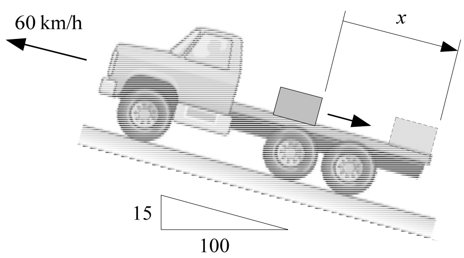

Figure 18: Flatbed truck and crate.

QUESTION 3

A ball is released from rest relative to the lift in Figure 19 at a distance h_1 from the floor. The speed of the lift at the time of ball release is v_0. Determine the bounce height h_2 of the ball (a) if v_0 is constant and (b) if an upward lift acceleration $a = g/4$ begins at the instant the ball is released. The coefficient of restitution for the impact is e.

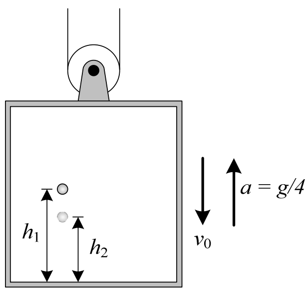

Figure 19: Lift with ball and acceleration indicated.

Tutorial Sheet 9

QUESTION 1

Question 8.1 from Lee.

Answer: 3.3333 mm from the centre of the rod.

QUESTION 2

Rotation of the lever *OA* in Figure 20 is controlled by the motion of the contacting circular disc whose centre is given a horizontal velocity *v*. Determine the expression for the angular velocity ω of the lever *OA* in terms of *x*.

HINT: Determine r in terms of x and the angle of OA. Then differentiate with respect to time. What is dr/dt? Answer: $\omega = \frac{v}{x\sqrt{(x/r)^2 - 1}}$

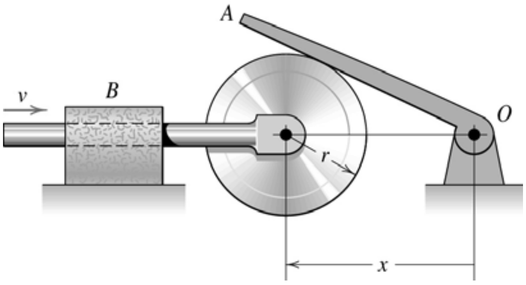

Figure 20: Lever and disc mechanism.

QUESTION 3

A 200 kg space vehicle is travelling with velocity 100**i** m/s when explosive charges separate it into two parts, one of 50 kg and one of 150 kg. The 150 kg part travels onwards with velocity 120**i** + 10**j** + 20**k** m/s. What is the velocity of the 50 kg part after the separation?

Answer: 40**i** − 30**j** − 60**k** m/s

QUESTION 4

A 200 kg space vehicle travelling with the velocity $\mathbf{v}_0$ = 100**i** m/s passes through a fixed point *O* at *t* = 0. Explosive charges then separate the vehicle into two parts, *A*, *B* of mass 150 kg and 50 kg.
Knowing that, at *t* = 10 s, the position of part *A* is observed at (1000, −200, 200), where the coordinates are expressed in metres relative to *O*, determine the position of part *B* at that time.

Answer: (1000, 600, −600)

QUESTION 5

Question 7.10 from Lee.

Answer: i) $\tan^{-1}0.25$, ii) $\tan^{-1}0.33333$

QUESTION 6

Determine the minimum speed *v* and the corresponding angle θ in order that the motorcycle in Figure 21 may ride on the vertical wall of a cylindrical track following a constant horizontal path around the track. The effective coefficient of friction between the tyres and the wall is 0.70. Assume that all forces and accelerations lie in the plane of the figure

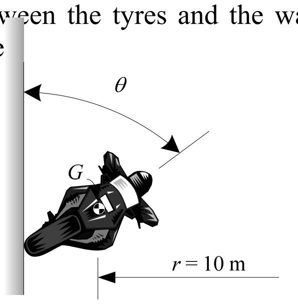

Figure 21: Motorcyclist and vertical wall.

Answer: v = 11.838 m/s, θ = 35°

Tutorial Sheet 10

QUESTION 1

The flywheel of a car can be modelled as a disc of diameter 0.2 m and thickness 0.02 m from which another concentric disc of diameter 0.1 m has been removed. The mass of the wheel is 3/4 kg.

The moment of inertia of a solid disc of radius r and mass m about its axis is $\frac{1}{2}mr^2$.

i. Calculate the moment of inertia of the flywheel about its axis.

The starter motor of the engine makes the flywheel rotate at 300 rad/s after 3 seconds starting from rest.

ii. Find its angular acceleration (assumed constant).
iii. Find the average torque required to accelerate the flywheel.
iv. Find the kinetic energy of the flywheel after 3 seconds.

Answers: i) 0.0046875 kg·m^2, ii) 100 rad/s^2, iii) 0.46875 N·m, iv) 210.9375 J

QUESTION 2

Question 20.13 from Lee. The moment of inertia of a solid disc of radius r and mass m about its axis is $\frac{1}{2}mr^2$.

Answer: ii) $T_1 - T_2 = Mg/8$

QUESTION 4

The 50 kg dynamics instructor in Figure 23 is demonstrating the principles of angular momentum in class. She stands on a freely rotating platform with her body aligned with the vertical platform axis. With the platform not rotating, she holds a modified bicycle wheel so that its axis is vertical. She then turns the wheel axis to a horizontal orientation without changing the 500 mm distance from the centreline of her body to the wheel centre, and the students observe a platform rotation rate of 0.25 rad/sec. If the rim-weighted wheel has a mass of 10 kg and a centroidal radius of gyration $\bar{k}$ = 200 mm, and is spinning at a fairly constant rate of 5 rad/s, estimate the mass moment of inertia I of the instructor (in the posture shown) about the vertical platform axis.

Note that the mass moment of inertia of the wheel about an axis through its mass centre and normal to the plane of the wheel (I_{zz} in the diagram below) is twice the moment of inertia about an axis through the mass centre and in the plane of the wheel (e.g. I_{xx} in Figure 22).

Answer: I = 3.45 kg·m^2

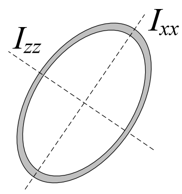

Figure 22: Axes of rotation of wheel.

Figure 23: Instructor with spinning wheel.

Tutorial Sheet 11

QUESTION 1

The spacecraft in Figure 24 is spinning with a constant angular velocity ω about the z-axis at the same time that its mass centre O is travelling with velocity v_0 in the y-direction. If a tangential hydrogen-peroxide jet is fired when the craft is in the position shown, determine the expression for the absolute acceleration of point A on the spacecraft rim at the instant the jet force is F. The radius of gyration of the craft about the z-axis is k, and its mass is m.

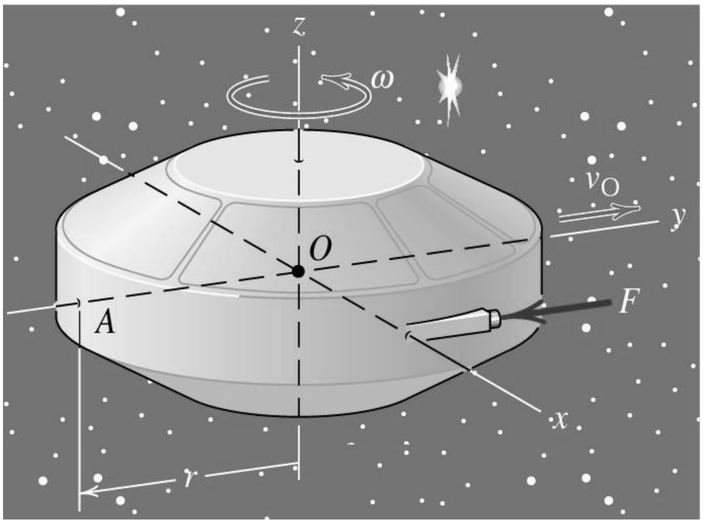

Figure 24: Spacecraft with direction of force indicated.

Answer: $\mathbf{a}_A = -\dfrac{Fr^2}{mk^2}\mathbf{i} - \left(\dfrac{F}{m} - r\omega^2\right)\mathbf{j}$

QUESTION 2

Horizontal motion of the piston rod of the hydraulic cylinder in Figure 25 controls the rotation of link OB about O. The velocity of A is a constant v_A = 2m/s. OB is horizontal and AB is vertical at the instant shown. The angular velocity and angular acceleration of OB are 0 rad/s and 20 rad/s^2 CW respectively for this instant. The angular velocity and angular acceleration of AB are 10 rad/s CW and 0 rad/s^2 respectively. The mechanism is positioned in a horizontal plane. The masses of the uniform rods AB and AB are 10kg and 5kg respectively. The moments of inertia of the rods about their mass centres are I_{OB} = 0.005 kg·m^2 and I_{AB} = 0.04 kg·m^2. Determine the magnitude of the reaction force at O at the instant shown.

Answer: $\sqrt{50^2 + 1^2}$ N

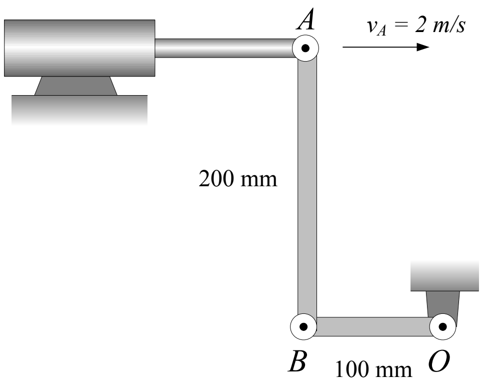

Figure 25: Mechanism.

QUESTION 3

The wheel and its hub have a mass of 30 kg with a radius of gyration about the centre of 450 mm. A cord wrapped securely around its hub is attached to the fixed support, and the wheel is released from rest on the incline. If the coefficients of static and kinetic friction between the wheel and the slope are 0.40 and 0.30, respectively, calculate the acceleration *a* of the centre of the wheel. First prove that the wheel slips.

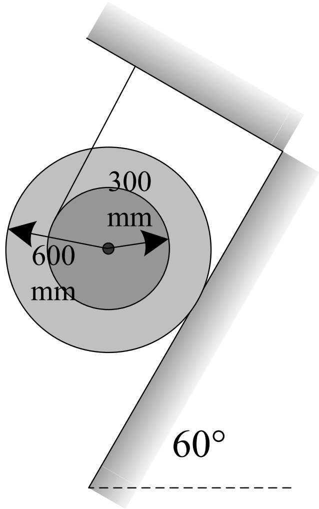

Figure 26: Wheel rolling down slope.

Answer: a = 1.256 m/s^2