	Financial Mathematics	
MS318	**Tutorial Sheet**	**3**
	Investment Appraisal	

3.1 A loan of €2 million produces an annually payable annuity of €250,000 for 25 years, where each payment is made in arrears. Assume that the rate of interest on borrowed money is $7\frac{1}{2}\%$ per annum, whilst that on money invested is 6% per annum. Find:

(a) the *payback period* of the project;

(b) the *discounted payback period*;

(c) the accumulated profit on the investment after 25 years.

3.2 A consortium takes over a failing business at an initial purchase price of €2.2 million. A thorough restructuring of the business' operations in the consortium's first year of management costs a further €750,000, payable continuously.

The consortium expects to accrue operating losses of €150,000 in the first year and €50,000 in the second year. However, it is projected that, in the following year and thereafter, the business will make profits of €350,000 per annum. These profits will pertain for 25 years. All cashflows are assumed to be payable or receivable *continuously*.

The venture is financed by bank loans, on which there is an effective rate of interest of 7%. Any funds which accumulate after the loan has been repaid are lodged in an account earning $6\frac{1}{2}\%$ per annum.

(a) Find the discounted payback period of the project.

(b) Find the accumulated amount in the account at the end of the 27 years of the project.

3.3 A company has embarked on the following project. It is to make an initial investment of three payments each of €105,000. The first is due at the start of the project, the second six months later and the third due one year after the start of the project. After 15 years, it is assumed that a major refurbishment of the business will be required, costing €200,000.

No income will be generated by the project in year 1, €20,000 received continuously in year 2, €23,000 in year 3, €26,000 in year 4 and €29,000 in year 5. Thereafter, the income is expected to increase by 3% per annum (compound) at the start of each year. The cash flow in each year is to be received at a constant rate. The project stops generating revenue at the end of its 30^{th} year.

(a) Calculate the NPV at a rate of interest of 8% per annum effective.

(b) Show that there is no discounted payback period within the first 15 years, assuming an effective rate of interest of 8% per annum.

(c) Calculate the discounted payback period for the project, assuming an effective rate of interest of 8% per annum.

3.4 Suppose that a fund is valued at €1.5 million on 1 January 1999, and income is received continuously for two years at a rate of €125,000 per annum. Find the money-weighted rate of return (MWRR) if the fund has value €2 million on 31 December 2000.

3.5 An investment fund is valued at €1.8 million on 1 January 2001 and at €1.9 million on 31 March 2001. On the same date, €400,000 is paid into the fund. The value of the fund is €2.6 million on 31 December 2001. Calculate the time-weighted rate of return (TWRR) for the year. Is it equal to the linked internal rate of return (LIRR)?

3.6 Suppose that the force of interest is given by

$$\delta(t) = 0.03 - 0.001t + 0.002t^2, \quad 0 \leq t \leq 5.$$

(a) Calculate the accumulated value at time 5 of €1,000 invested initially, together with an investment of €500 at time 2.

(b) What is the equivalent constant force of interest on the transaction?

3.7 The force of interest at time t is given by

$$\delta(t) = \begin{cases} 0.05 + 0.01t, & 0 \leq t \leq 3, \\ 0.08 - 0.02t, & 3 < t \leq 6. \end{cases}$$

Find the effective rate of interest per unit time on an investment made at time 0 and held until time 6. Express your answer as a percentage, correct to 2 decimal places.

3.8 The force of interest $\delta(t)$ is a function of time and, at any time t, measured in years, is given by

$$\delta(t) = \begin{cases} 0.06 + 0.005t & 0 \leq t < 4 \\ 0.12 - 0.01t & 4 \leq t < 6 \\ 0.06 & 6 \leq t \end{cases}$$

(a) Derive expressions, in terms of t, for the accumulated amount at time t of an investment of 1 at time $t = 0$.

(b) Calculate the value at time $t = 0$ of €100 due at time $t = 5$.

(c) Calculate the accumulated value at time $t = 12$ of a payment stream, paid continuously from time $t = 4$ to $t = 6$, under which the rate of payment at time t is $\rho(t) = 12 - t$.