MS308 - STOCHASTIC MODELLING

Continuous Assessment, November 2005

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INSTRUCTIONS: ANSWER ALL QUESTIONS

QUESTION 1

State the Markov property and use it to prove the € Chapman-Kolmogorov equations for a process with a discrete state space. Hence derive the matrix form of these equations; solve the equation in the case of a discrete-time homogeneous chain.

[18 Marks]

QUESTION 2

Define the following: stationary process, stationary probability distribution. Prove that a Markov chain with stationary probability distribution π is a stationary process if started with initial condition π (i.e. $\mathbb{P}[X_0 = j] = \pi_j, \quad j = 1, 2, 3, ...$).

[20 Marks]

QUESTION 3

Consider the Markov chain with transition matrix

$$\mathbf{P} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$$

Draw the corresponding transition graph. Is the chain irreducible, is it aperiodic? Justify your answers. Find the stationary distribution of the chain. What can you say about $\lim_{n\to\infty} P^n$?

[18 Marks]

QUESTION 4

Define the following: communicating classes, periodic state. Prove that periodicity is a class property (including the value of the period). Is it possible for a chain to have some periodic states and some aperiodic states? If so, supply an example; if not, prove your assertion.

[18 Marks]

QUESTION 5

Give a mathematical definition of $f_{ij}(n)$, the probability that the first visit to state j starting from state i occurs at step n.

Prove the formula

$$f_{ij}(n) = \sum_{m=1}^{n} f_{ij}(m) p_{jj}(n-m)$$

where $p_{ij}(k)$ is the k-step transition probability of the Markov chain.

[18 Marks]

QUESTION 6

Define the following terms: recurrent state, positive-recurrent state, null-recurrent state. State a result on $\lim_{n\to\infty} p_{jj}(n)$ when j is a recurrent state.

[8 Marks]