DUBLIN CITY UNIVERSITY

SEMESTER ONE REPEAT EXAMINATION 2005

MODULE:	MS308 Stochastic Modelling
COURSE:	B. Sc. in Financial and Actuarial Mathematics, B. Sc. in Mathematical Sciences
YEAR:	3
EXAMINERS:	Mr. P. Cooper, Dr. R. Gray, Prof. T. C. Hurley, Prof. E. Buffet (ext. 5287)
TIME ALLOWED:	2 hours
INSTRUCTIONS:	Candidates who are registered for Actuarial Exemptions must answer **all** four questions. Candidates who are **not** registered for exemptions should attempt any **three** out of four questions. Each question carries 25 marks.
REQUIREMENTS:	Candidates should provide their own electronic calculators. Mathematics tables will be provided by the university; Actuarial Tables are **not** required.

THE USE OF PROGRAMMABLE OR TEXT STORING CALCULATORS IS EXPRESSLY FORBIDDEN

DO NOT TURN OVER THIS PAGE UNTIL INSTRUCTED TO DO SO.

QUESTION 1

(a) When is a state of a Markov chain said to be recurrent? Establish a system of linear equations relating the transition probability $p_{ij}(n)$ to $f_{ij}(n)$, defined to be the probability that the first visit to state j starting from state i takes place at step n.

[6 marks]

(b) Using the above equation, prove that state j is transient if $\sum_{n=1}^{\infty} p_{jj}(n) < \infty$.

[9 marks]

(c) Prove that state j is recurrent if $\sum_{n=1}^{\infty} p_{jj}(n) = \infty$.

[10 marks]

QUESTION 2

The members of a health insurance scheme are classified as nett contributors or nett beneficiaries; a member who is a contributor in one period becomes a beneficiary in the next if he or she becomes seriously ill, and this happens with probability 0.2. The probability of a serious illness continuing into the next period is 0.15. The following rules are in operation:

No one is entitled to be a nett beneficiary for more than three successive periods; following this (and assuming that the illness persists) there must be at least one contributory period before any further period as beneficiary.
(a) Construct a Markov chain to model this health scheme, introducing if necessary various classes of beneficiaries and contributors (a five state model is suggested). Draw the transition graph and write down the transition matrix of the chain. Is the above Markov chain irreducible? Is it aperiodic? Justify your answers.

[7 marks]

(b) Calculate the stationary probability distribution of the chain.

[9 marks]

(c) What is the proportion of nett beneficiaries among the membership in the stationary regime? Let b (respectively c) be the average gross payout per beneficiary (respectively per contributor) per period; this means that the nett payments are $b-f$ and $c-f$ respectively where f is the membership fee per period (assumed to be uniform over members and over time). Suppose that the overheads per member and per period are d, how should b, c, d and f be related if the scheme is to be viable?

[9 marks]

QUESTION 3

(a) Define the Poisson process and write down its generator. Is the process conservative, is it irreducible?

[6 marks]

(b) Solve the forward equations for the Poisson process.

[12 marks]

(c) Write down the backward equations for the Poisson process.

[3 marks]

(d) Claims reach an insurance company according to Poisson process. On average, 100 claims reach the company every month. What is the probability that at least one claim is received on a given day (assume a 30 day month).

[4 marks]

QUESTION 4

(a) State the Markov property, and use it to derive the Chapman-Kolmogorov equations for a continuous-time Markov process with a discrete state space. Write down the form that the equations take for a time-homogeneous process.

[6 marks]

(b) Derive the backward and forward equations from the above.

[4 marks]

(c) Define holding times and establish the probability distribution of the first holding time.

[10 marks]

(d) Describe in detail (but without proof) the behaviour of a Markov jump process in terms of its associated jump chain and holding times.

[5 marks]