Advertising effectiveness and spillover: simulating strategic interaction using advertising

25th International Conference of the System Dynamics Society
Boston, Massachusetts
29th July to 2nd August, 2007

Malcolm Brady
Business School
Dublin City University
Dublin, Ireland
malcolm.brady@dcu.ie

Abstract

This paper examines the strategic interaction between two firms competing using advertising. The simulation is based on the Cournot analytical duopoly model. The paper discusses the analytical model and then discusses the simulation model. The paper discusses the ways in which the simulation model is different, usually for pragmatic reasons, to the analytical model. The paper shows results of the model for a number of competitive scenarios. These scenarios demonstrate that small changes in model parameters can significantly alter firm and industry behavior. The paper demonstrates that positive spillover (cooperation) reduces the impact of advertising and negative spillover (predation) increases the impact of advertising.

Keywords: strategic interaction, duopoly, advertising, spillover, simulation

Introduction

This paper depicts a dynamic model of two firms competing using advertising. The paper discusses an analytical model based on the Cournot duopoly model and then goes on to discuss the simulation model built to examine the dynamic behavior of this model as advertising alters demand. The model is set up in system dynamics style using the Powersim simulation software package. The objective of the model is to simulate dynamic behavior when two firms compete using advertising.

The basic duopoly model leads to an equilibrium which can be determined analytically (Dixit, 1979); this basic model does not demonstrate any dynamic behaviour. Introducing advertising into the model allows firms endogenously alter demand which does invoke dynamic behavior but is analytically intractable. Simulation is a means of tackling such problems. Indeed, simulation is a research technique that is becoming more broadly recognised in the management literature (Davis et al, 2007).

Combining economic models and the system dynamics approach is not new. Radzicki and Sterman (1994) have examined duopoly using an approach that combines system dynamics, evolutionary economics and the learning effect. Forrester (2003) discussed system dynamic modeling of economic systems in 1956, including a brief review of an advertising model. Graham and Ariza (2003) present a model that optimizes allocation of firm advertising expenditure using a simulated annealing approach. Sterman et al (2007) use an approach that combines duopoly theory with the behavioural theory of the firm. A contribution of this paper is to formally model the game theoretic Nash equilibrium using a system dynamics approach.

The analytical model

Profit is the ultimate measure of firm performance and I start with the profit function which is determined from the accounting identity:

$$\Pi_i = p_i q_i - c_i q_i \ , \ i = 1,2 \qquad \ldots(1)$$

where p represents unit price, q represents quantity sold and c represents unit variable cost; subscript i represents one firm and subscript j represents its rival. The firm is assumed to be in control of its costs. A simple cost function is used in equation (1) as more complex cost functions significantly increase the complexity of the algebra when determining the Nash equilibrium quantities. Fixed costs are assumed to be zero.

The market is assumed to determine price and quantity according to the analytical model based on the work of Dixit (1979) and Bowley (1924). Their work is founded on neoclassical economic principles and is largely a theory of markets. They suggest that price can be modeled as a downward sloping function of quantity and in a two-firm situation represent this as follows:

$$p_i = a_i - bq_i - dq_j \qquad i = 1,2; j = 3 - i \qquad \ldots (2)$$

where p is price, a is the reservation price, b is the own-price effect, d is the cross-price effect, and q is quantity placed on the market. Subscript i refers to the firm and subscript j to its rival. The reservation price can be imagined as the highest price that is likely to be paid for the good. The own-price effect represents the impact on price of placing additional goods on the market. The cross-price effect represents the impact on the firm's price when its rival places additional goods on the market. The ratio of cross-price effect to own-price effect represents the level of differentiation between the two products: a value of one implies that the products are perfect substitutes; a value of zero implies that the products are entirely differentiated, each product being in effect a monopoly; a value of between zero and one represents differentiated products and the value indicates the extent of differentiation.

The model is in the Cournot (1838) tradition in that price is represented as a function of quantity ie. firms are assumed to put a certain quantity onto the market and the marketplace then determines a price for the good. A model could be created for the inverse situation where a firm sets price and the marketplace determines quantity sold

(Bertrand, 1883); however I leave the reporting of the simulation of that situation to another paper.

The Dixit approach is to determine the Nash (1951) equilibrium quantities for the two-firm competitive situation: the equilibrium represents a pair of quantities such that neither firm has an incentive to shift from their choice of quantity, given the choice of the other firm. This is achieved by differentiating each of the two profit functions (1) with respect to quantity, setting the result to zero to determine a maximum, and solving the two resulting equations (usually referred to as 'reaction functions') simultaneously to determine the two quantities. This process results in the following formal expression of the Nash equilibrium quantities when products are differentiated:

$$q_i = \frac{2a_i b - a_j d - 2bc + cd}{4b^2 - d^2} \qquad i = 1,2; j = 3 - i \qquad \ldots(3)$$

Price is then be determined from equations (2) above and profit in turn from equations (1). Equations 1, 2 and 3 set up the market behaviour of the two firms and give the resulting profits for the two firms. The model so far is static in that the equilibrium values will pertain for all time. I now introduce advertising into the model. Advertising, paid for from firm profits, will alter the demand functions of the two firms and so introduce a dynamic element to the model. Simulation is a means by which the system dynamics can be observed.

Endogenising advertising into the model has three impacts. Firstly, the effect of advertising on demand must be determined. Secondly, the amount of advertising selected by each firm must be determined. Thirdly, the cost of advertising must be included in the model.

The effect of advertising is to stimulate demand in some fashion. In terms of the model outlined above advertising alters the demand function: advertising may shift the demand function to the right, change the slope of the demand function, or both. In this paper I consider the case where advertising shifts the demand function to the right which can be broadly regarded as increasing demand for the firm's product. In modeling terms such a shift means that advertising increases the value of the reservation price, *a*.

I follow Friedman's (1983) two suggestions: that the impact of advertising is cumulative and that interfirm effects exist. The cumulative impact of advertising is taken into account in that once the demand curve shifts to the right it stays shifted and does not revert to its original position. Further advertising of course may cause it to shift further to the right. The interfirm aspect of advertising is modeled by including a spillover effect ie. advertising by one firm may also increase demand for its rival's product. I adapt Friedman's expression to determine the impact of advertising:

$$\Delta a_i = \varphi_i A_i + \rho \varphi_j A_j, \quad i = 1,2, \quad j = 3 - i \qquad \ldots. (4)$$

where a is the reservation price (the intercept of the inverse demand function with the vertical axis), A is the amount of advertising, φ represents the effectiveness of firm advertising and ρ represents the extent of advertising spillover. Δa represents the change in demand due to advertising of both firms. In system dynamics terms equation (4) becomes a rate equation that determines the rate of change of the stock variable *a* due to advertising. Note that advertising spillover can be negative as well as positive – if the spillover parameter is given a negative value then advertising by one firm reduces demand for the other firm.

To determine the optimal amount of advertising for a firm I use Dorfman and Steiner's (1954) condition. In particular I use Ferguson et al's (1993) representation of the condition which states that optimal advertising is a proportion of revenue in the ratio of advertising elasticity of demand to price elasticity of demand; (recall that revenue R = pq):

$$A_i = R_i \frac{\eta_{Ai}}{\eta_i} \qquad i = 1,2 \qquad \ldots(5)$$

Advertising elasticity of demand can be shown (Author name withheld, 2004) to be:

$$\eta_{Ai} = \frac{A_i}{q_i} . \frac{\varphi_i (b - \rho d)}{b^2 - d^2} \qquad i = 1,2 \qquad \ldots(6)$$

Note that advertising effectiveness and advertising spillover both influence advertising elasticity. Note also that substituting equation (6) for advertising elasticity into equation (5) leads to advertising appearing on both sides of equation (5). To avoid circularity I use the delay feature in Powersim to specify the amount of advertising in the previous period as a proxy for current period advertising when determining advertising elasticity according to equation (6).

Price elasticity of demand can be shown (Author name withheld, 2004) to be:

$$\eta_i = \frac{p_i}{q_i} . \frac{b}{b^2 - d^2} \qquad i = 1,2 \qquad \ldots(7)$$

It is relatively straightforward to include the cost of advertising into the model. Advertising is a cost to the firm and reduces profit accordingly. The profit identity (equation 1) now becomes:

$$\Pi_i = p_i q_i - c_i q_i - A_i \ , \qquad i = 1,2 \qquad \ldots(8)$$

Note that the usual microeconomic assumption with respect to production applies to this model: it is assumed that production can be increased or reduced instantaneously in order to produce the amount of goods required. For this reason no formal production function is included in this model. This assumption implies that there exist no constraints on the instantaneous supply of human and technological resources. Real systems do of course have lags in providing resources and gearing up the production system to meet demand change. It is well known that lags lead to oscillatory behavior and many system dynamics models focus specifically on these lags (for example, see Sterman, 1989). I do not do so here but combining a lagged production model with the Cournot model described above could be a fruitful area for further research.

The simulation model

The simulation model was created using the Powersim simulation software package. The model used in this paper is a subset of a larger model that can be used to simulate a tilt as well as a shift of the demand function and can also simulate organic and cyclic change in demand. The full model also simulates the profit and loss statement for each firm and includes variables to model fixed costs, depreciation, tax, and dividends; however relevant parameters are set to zero and so these variables are redundant for the simulations discussed in this paper.

The model subset used in this simulation contains five stock variables, five flow variables, ten auxiliary variables and eight parameters, listed in appendix 1. Not untypically, the number of variables in the simulation model is greater than the number of variables in the analytical model. Firstly three stock variables are used to record retained earnings over time: one accumulator (capital) records positive retained earnings ie. profits. Because it is useful to record losses separately, a second accumulator records accumulated losses over time. This allows different rules to apply when dealing with losses. In this simulation losses are assumed to be met by short term borrowing and incur an interest penalty. If they were to be accumulated in capital then losses would reduce the capital, and hence the productive capacity, of the firm. A third accumulator records the net present value of profits over time; this stock variable is used solely for reporting reasons. Secondly, to separate out the impact of advertising on demand from the impact of organic growth or decay in demand two separate accumulators are used to record the reservation price. The stock variable - demand_reservation_price_no_advertising - records the value of the reservation price as it changes organically. This variable is not strictly required for this paper as organic growth or decay is not modeled here; however it has been built into the model to allow organic change be examined as part of a future research project. The stock variable - advertising_impact_on_reservation_price - accumulates the change in demand over time due to advertising. The five rate variables mirror the use of these five stock variables.

Four additional parameters are included in the simulation model for practical reasons. Firstly, the parameter - advertising_weighting_factor - is a switch that allows advertising to be turned on or off at one go without having to alter a series of parameter values. Secondly, the rule for determining optimal advertising can occasionally select very high amounts of advertising. To place an upper limit on advertising a new parameter - advertising_rate - was introduced into the model to limit advertising to a proportion of revenue; for this paper advertising was limited to 50% of revenue as it is unlikely that any real firm would invest in advertising beyond this limit. Thirdly, an interest rate parameter is included to allow any losses incurred by firms during the simulation to be charged for. Fourthly, a discount rate parameter is included to allow net present value of retained earnings be determined; this is required for reporting reasons only.

The model contains a number of feedback loops that centre on the impact of advertising on the demand intercept (reservation price). Advertising influences reservation price which in turn influences equilibrium quantity selected, and quantity influences advertising. This generates the reinforcing feedback loop R1 shown in figure 1. This loop can be traced by tracking the variables in equations 3, 4 and 5. However, the complexity in the model arises because quantity influences advertising in a number of ways. Advertising elasticity of demand also positively influences advertising through equation (5) and quantity negatively influences advertising elasticity through equation (6). This generates the balancing feedback loop B1 also shown in figure 1.

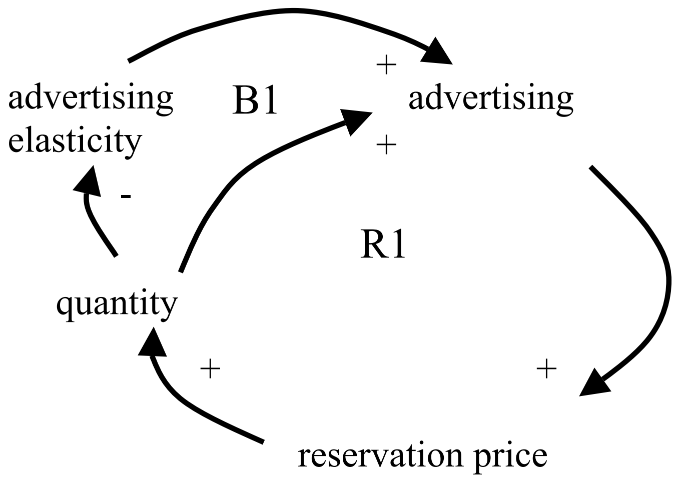

Figure 1: Reinforcing loop R1 and balancing loop B1

Advertising is also influenced by price elasticity of demand according to equation (5). This generates three additional feedback loops, two reinforcing and one balancing as shown in figure 2. An increase in price elasticity decreases advertising which generates reinforcing loop R2 and R3. Balancing loop B2 is generated when an increase in reservation price increases quantity which decreases price which decreases advertising which decreases reservation price. Note that there is an additional reinforcing loop implied by the recursion inherent in equations (5) and (6). In the simulation model this is treated as a delay in the influence of advertising on advertising elasticity of demand. I will refer to this loop as R4. This is the only loop that does not include quantity and reservation price. The simulation models a situation where two firms are in competition with each other. The use of arrays to simulate competition means that two sets of the above feedback loops exist, one for each firm.

The two firms interact in two ways: in determining the Nash equilibrium quantities as shown in equation (3) and in determining the impact of advertising due to spillover as shown in equation (4). The interaction generated by equation (4) creates an additional set of interaction loops similar to those given in figures 1 and 2 but where all variables except reservation price refer to the rival firm. The interaction generated by equation (3) creates a set of loops similar to those of figures 1 and 2 but with signs reversed. This is because the impact of an increase in the rival's reservation price is to decrease firm quantity.

Where advertising is predatory the effect of spillover is negative, ie. firm advertising reduces rival's demand. The impact of an increase in advertising is to reduce rival reservation price. This has the effect of reversing once again the signs of the interaction loops discussed in the previous paragraph to their original values as shown in figures 1 and 2.

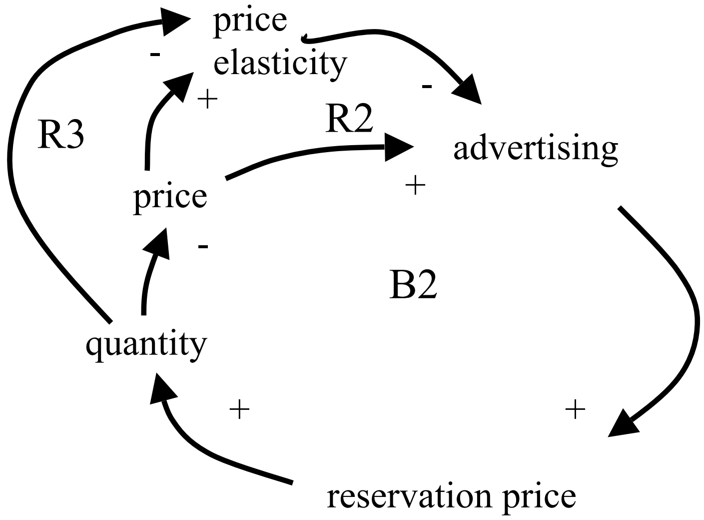

Figure 2: Reinforcing loops R2 and R3 and balancing loop B2

Simulation Results

The simulation model was used to examine competitive behavior between two firms when advertising is used as a competitive weapon. The other competitive weapon implicitly used in the model is quantity; however the quantity decision is not discretionary – it is always optimal in the Nash equilibrium sense. This paper assumes that firms have some discretion over advertising and so several advertising-relevant scenarios are examined. Different scenarios are generated simply by changing the parameters controlling advertising effectiveness and advertising spillover. I will now discuss three main scenarios: firstly I examine the baseline case when neither firm advertises; then I examine the impact of altering advertising effectiveness; then I examine spillover of the impact of advertising onto rival firm demand.

In order to carry out the simulation the model must be initialised by assigning values to parameters. The scenarios examined in this paper refer to a high volume low price product, for example a book, an item of clothing, or tin of paint that costs \$8 to produce. Unit variable cost *c* for both firms is therefore set at \$8. The initial reservation price *a* is set at \$25; this means that the highest price likely to be achieved for the product is \$25 even if the good was scarce. Products are assumed to be differentiated and so own-price effect *b* is set at 0.0001 and cross-price effect *d* at 0.00005. This implies that 10,000 extra units of product placed on the market will reduce the price of the product by \$1. It also means that the products are differentiated and the effect of a rival placing a quantity on the market affects firm price only half as much as the firm itself placing that same quantity on the market. Results for the baseline case where neither firm advertises are shown in figure 4a. This gives the Cournot-Nash equilibrium for the industry and it is clear that it remains constant for all time.

Advertising is now introduced into the model. For the first scenario only one of the two firms is assumed to advertise. This is achieved by setting the advertising effectiveness parameter for firm one to 0.000015 which means that $66,667 of advertising raises the reservation price by $1. At this relatively high level of advertising effectiveness the Cournot-Nash equilibrium evolves so as to drive the non-advertiser out of the market as shown in figure 4b. Clearly the reinforcing loops dominate behavior.

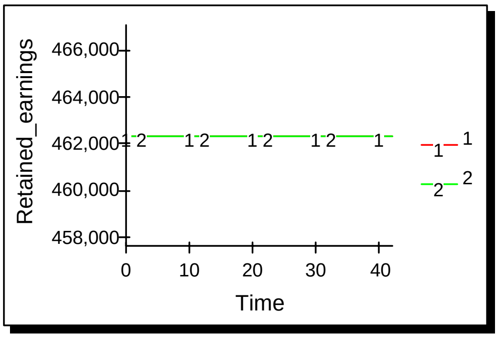

Figure 4a. No advertising

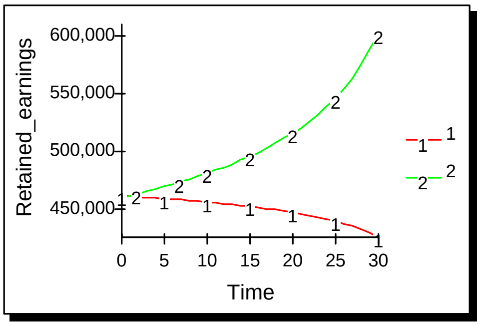

Figure 4b. Firm 1 advertises effectively φ1=0.000015; φ2=0

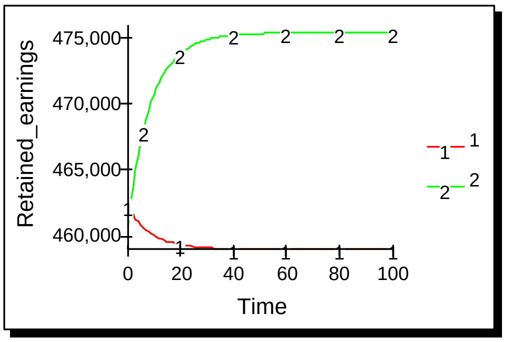

Fig. 4c. Firm 1 advertises ineffectively φ1=0.000013; φ2=0

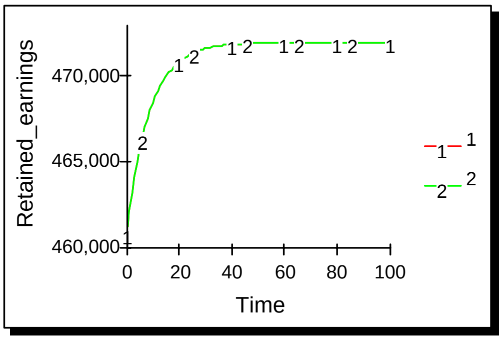

Fig. 4d. Both firms advertise ineffectively $\varphi1=\varphi2=0.000013$

As the firm becomes less good at advertising and advertising effectiveness reduces a bifurcation in industry behavior occurs. Trial and error experimentation determined the threshold at which industry behavior bifurcated to be at an advertising effectiveness of 0.0000139. Below this threshold level the industry evolves to a stable Cournot-Nash equilibrium with both firms remaining in the market with little advantage accruing to the advertising firm; above this threshold the advertising firm dominates the non-advertiser and forces it out of the industry. Figure 4c shows results for a scenario where advertising effectiveness is set at 0.000013 ie. below the threshold level; clearly the balancing loops dominate behavior in this scenario.

Next I allow both firms advertise. The bifurcation in industry behavior similarly occurs when both firms advertise as shown in figure 4d where advertising effectiveness for both firms is set at 0.000013. When advertising effectiveness is above the threshold level for both firms then simulation experiments show that both firms grow exponentially. These results support the empirical observation that advertising takes place in some industries but not in others: ie. that advertising sometimes makes a difference to demand and sometimes does not. The results also show that advertising can be a very effective competitive weapon.

The above discussion has centred on advertising effectiveness. I now examine advertising spillover. Firstly I assume positive spillover ie. advertising is cooperative: advertising by one firm has a positive impact on its rival's demand. I start with the scenario situation modeled in figure 4b where one firm advertises effectively. I now set the advertising interaction factor ρ to 0.3 which means that the impact of a firm's advertising on its rival's demand function is 30% of that on its own demand function. The results in figure 5a show that, in contrast to those of figure 4b, advertising has little impact on demand: when only one firm advertises spillover has the effect of reducing the impact of advertising. This occurs because spillover increases the reservation price of the rival as well as the reservation price of the firm (equation 4) and this higher rival reservation price has the effect of reducing the optimal quantity selected by the firm (equation 3) leading to a reduction in the competitive advantage of the firm over its rival. Positive spillover also reduces the advertising elasticity of demand (equation 6) leading to less advertising being selected (equation 5) at the optimal point; less advertising leads to less of an increase in the firm's demand curve.

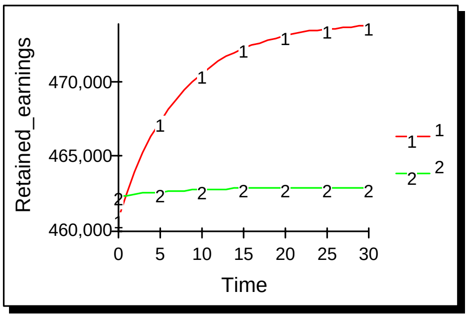

Figure 5a. Advertising spillover φ1=0.000015; φ2=0; ρ=0.3

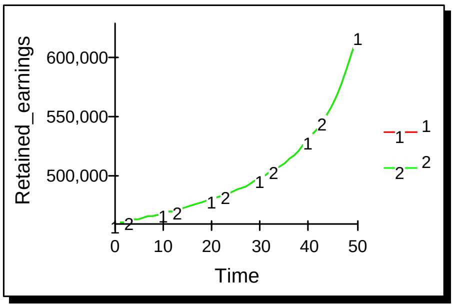

Figure 5b. Negative spillover φ1= φ2=0.000013; ρ= -0.3

Simulation experiments show that, when one firm advertises, spillover has the effect of increasing the bifurcation threshold level of advertising effectiveness at which reinforcing loops dominate balancing loops. This suggests that as advertising tends towards being a public good firms will be less inclined to advertise unilaterally: firms will not advertise unless other firms in the industry also advertise. This is somewhat counterintuitive as spillover from rival firm advertising increases demand as shown in equation (4) and this should increase the rate of growth under the interaction reinforcing loop R1. However, spillover also reduces advertising elasticity of demand as shown in equation (6) decreasing the amount of advertising selected (equation 5) and increasing the influence of balancing loop B1.

An interesting and non-intuitive result is that when both firms advertise cooperatively spillover raises the bifurcation threshold level, and the higher the level of spillover the higher the threshold. This means that in order to gain from cooperative advertising firms must be more effective advertisers. Or, in other words, spillover reduces the impact of advertising.

When advertising is predatory, ie. spillover is negative, the bifurcation threshold level is lowered. Less effective advertisers can gain from advertising when advertising is predatory. Figure 5b shows results for the scenario shown in figure 4d except that the advertising interaction parameter is set at –0.3. Spillover is negative and reduces the demand for the rival firm. The results however show that industry behavior has switched: both firms now grow exponentially. Negative spillover has increased the effectiveness of advertising. These results suggest that predatory advertising may be more effective for firms than cooperative advertising.

Discussion and Conclusion

John Wanamaker, founder of the department store concept, famously said in 1886 that ‘I know that 50% of my advertising is wasted. I just don’t know which half’ (Black, 2003). This variability in response to advertising is usually put down to chance or stochastic effects. The results from this model demonstrate that sometimes the impact of advertising is strong and the firm grows spectacularly and sometimes the impact is weak and the firm does not grow. This model, however, is deterministic and the results are not due to chance or probability; nor are they due to one-off spikes or step changes in parameter values. They are due to system dynamic complexity and this complexity is inherent in the analytical equations. Different patterns of results occur as initial conditions are varied. The results above show that for some initial conditions reinforcing loops dominate and firms grow and for other initial conditions balancing loops dominate and firms come to an equilibrium. For yet other initial conditions one firm grows and one stagnates. The results demonstrate that threshold levels of advertising effectiveness exist where industry behavior bifurcates. The results also suggest that predatory advertising yields more benefit for a firm than does cooperative advertising.

From the point of view of feedback loops this is a relatively simple model with just one stock variable involved in all loops. However, the complexity in the model lies in the specification of the equations rather than in the loops. A further level of complexity lies in the interplay between array variables. This leads to the interesting situation where detail complexity invokes dynamic complexity (see Senge 1990 for a discussion of detail and dynamic complexity). However the dynamic complexity does not occur within a single simulation run but across different simulation runs.

The simulation model provides insight into a number of different competitive scenarios, the results for only some of which have been presented in this paper. Different choices of parameter values specifying different levels of advertising effectiveness and spillover led to different loops dominating the dynamic behavior of the system. High levels of advertising effectiveness lead to the reinforcing loops dominating and low levels lead to balancing loops dominating. Some interesting situations occurred where the reinforcing loop dominating one firm led to explosive growth for that firm and decay for the rival firm, ultimately driving the rival out of business.

The Nash equilibrium is a game theoretic construct and an interesting aspect of the model is that it simulates a game theoretic system as it evolves over time. The diagrams in figures 4 and 5 above demonstrate the evolution of the Nash equilibrium as it changes over time due to self induced system changes.

Analyzing the loop structure of the model for this paper provides useful insight into the dynamic behavior of the system. Constructing the causal loop diagrams separates out the various loops implicit in the equations. It also identifies the loops that are reinforcing and those that lead to equilibrium. The results show that under different initial conditions different loops dominate the behavior of the system.

Some practical points are worth referring to. Using arrays requires careful coding to ensure mistakes do not occur especially if, as in this simulation model, some formulae are long and complex.

Converting the analytical model into a simulation model required the introduction of additional variables for pragmatic reasons. For example, the rule for determining optimal advertising can occasionally select very high amounts of advertising. To place an upper limit on advertising a new parameter (advertising_rate) was introduced into the model that limited advertising to a proportion of revenue.

Use of the camera/snapshot feature of Powersim allowed duplicate icons for variables to be introduced into the model, reducing the number of relationship lines and the spaghetti-like look of a complex model. Distinct sections of the model can be independently represented graphically; this makes the model more comprehensible and aids the process of tracking down numerical errors during testing. However, a disadvantage of this approach is that it makes it more difficult to recognise the feedback loops inherent in a model.

The model discussed in this paper makes the standard microeconomic assumption that production resources can be instantaneously increased or decreased to meet demand. In practice of course resources cannot be ramped up or down instantaneously and much managerial resources are devoted to ensuring resources match demand requirements. Indeed much system dynamic modeling of firms specifically examines the pipeline of resources that together form the firm's supply chain. An avenue for future research is to enhance the advertising duopoly model suggested in this paper to include a lead time when increasing the resources required for production. Introducing such a lag or delay into the model which would inevitably stimulate new dynamic behavior in the system.

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Appendix 1

Simulation Model Variables

Stocks

Accumulated_loss
Accumulated_profit_npv
Advertising_impact_on_demand_reservation_price
Capital
Demand_reservation_price_no_advertising

Flows

Advertising_impact_on_reservation_price_rate
Change_in_reservation_price
Retained_loss
Retained_profit
Retained_profit_npv

Auxiliary

Advertising	(variable A in the analytical model)
Advertising elasticity of demand	(variable η_A in the analytical model)
Cross_price_effect	(variable d in the analytical model)
Demand_reservation_price	(variable a in the analytical model)
Depreciation	(not used for this paper)
Dividend	(not used for this paper)
Earnings_before_tax	(redundant for this paper)
Gross_income	(redundant for this paper)
Interest	(redundant for this paper)
Net_income	(redundant for this paper)
Net_operating_income_or_EBIT	(redundant for this paper)
Price_elasticity_of_demand	(variable η in the analytical model)
Retained_earnings	(variable Π in the analytical model)
Revenue	(variable R in the analytical model)
Tax	(not used for this paper)
Total_cost	(equivalent to TVC for this paper)
Total_variable_cost	(TVC = cq)
Unit_price	(variable p in analytical model)
Units_demanded	(variable q in analytical model)

Parameters

Advertising_effect_on_reservation_price	(advertising effectiveness φ)
Advertising_interaction_factor	(advertising spillover ρ)
Advertising_rate	(maximum percentage of revenue that can be used for advertising, set at 50%)
Advertising_weighting_factor	(to switch advertising on or off)
Discount_rate	(to determine net present value, set at 5%)
Dividend _retention	(set to zero for this simulation)
Fixed_production_cost	(set to zero for this simulation)

Interest_rate	(used to charge for losses, set at 7%)
Own_price_effect	(parameter b in analytical model)
Tax_rate	(set to zero for this simulation)
Unit_variable_cost	(parameter c in analytical model)