IMPACT OF ADVERTISING ON DUOPOLY COMPETITION

Competitive paper in Marketing track

POSTGRADUATE PAPER

Presented at

Irish Academy of Management Annual Conference,

Trinity College, Dublin

2nd – 3rd September 2004

Malcolm Brady

Dublin City University Business School

Dublin 9, Ireland

Tel. +353 1 7005188

Email malcolm.brady@dcu.ie

ABSTRACT

This paper proposes a model of optimal advertising for a duopoly. The advertising model is based on Dorfman and Steiner’s (1954) optimality condition. Advertising is introduced into a Cournot incentive duopoly model (Fershtman and Judd, 1987) extended to include differentiated products (Dixit, 1979). The paper develops the model analytically and then examines the behaviour of the firms in a duopoly using computer simulation. The paper examines the particular case where one firm adopts a low cost generic strategy and the other firm adopts a differentiation strategy (Porter, 1980 and 1985).
INTRODUCTION

This paper proposes a model of optimal advertising for a duopoly. The advertising model is based on Dorfman and Steiner’s (1954) optimality condition. Advertising is introduced into a Cournot incentive duopoly model (Fershtman and Judd, 1987) that has been extended to include differentiated products. Dixit (1979) suggests that two different forms of differentiation can take place. The first he refers to as absolute advantage and occurs when firms have different reservation prices. The second form occurs when cross price effect differs from own price effect. In this paper I assume that advertising shifts the demand function to the right by changing the value of reservation price. If this shifting is asymmetric then advertising leads to Dixit’s first kind of differentiation.

The paper is organised as follows. The paper develops the analytical model and then examines the behaviour of the firms in a duopoly using computer simulation. Firstly two base scenarios are examined where no advertising takes place; one scenario has symmetric costs and the other has asymmetric costs. Symmetric low and high levels of advertising are then introduced. Asymmetric advertising scenarios for both high and low levels of advertising are then examined. Finally asymmetric advertising and asymmetric cost scenarios are examined. This last scenario is used to examine the effectiveness of low cost versus differentiation strategies. The paper then concludes.

THE MODEL

The Dixit (1979) model assumes a linear downward sloping inverse demand function where price for a firm’s product is influenced by the quantity placed on the market by both firms. This is formalised in the expression:
\[ p_i = a_i - b_i q_i - d_i q_j \quad i = 1,2; \quad j = 3 - i \]

where \( p \) represents price, \( q \) represents quantity, \( a \) is the reservation price, \( b \) is the own price effect, and \( d \) is the cross price effect; subscripts \( i \) and \( j \) represent the two firms in the duopoly. When \( d \neq b \) firm products exhibit Dixit’s second kind of differentiation. Note that the reservation price is the intercept of the demand function with the vertical axis.

Interdependency of the two firms is represented by this pair of equations. In each time period a two stage game is played where in the first stage owners select optimal incentive factors and in the second stage managers select firm quantities at the Cournot equilibrium point (Fershtman and Judd, 1987; Cournot, 1838). Price is then determined according to the above expression and price, quantity, and cost determine profit for each firm using the usual accounting identity.

Advertising acts to increase the reservation price (parameter \( a \) in the above expression) and this shifts the inverse demand function to the right. This changing demand function gives the model its dynamic characteristic.

**SIMULATION**

I start this examination with a base case where firms have symmetrical demand functions and cost structure and neither firm advertises. For each firm reservation price is set at €25, own price effect is set at 0.0001, and cross price effect is set at 0.00005. Unit variable cost is set at €8 for each firm. Advertising effectiveness (\( \varphi \)) is set at zero for both firms. This scenario yields the symmetric steady state result shown in figure 1. Both firms place a quantity of 71,579 units on the market. At the
Cournot equilibrium point the market selects a price of €14.26 per unit and both firms make profit after tax of €268,986 each period for all time. The optimal incentive factor is 0.888 for each firm for all time.

Fig 1. Differentiated incentivised Cournot duopoly: \( \varphi_1 = \varphi_2 = 0; \ c_1 = c_2 = 8 \)

These results can be confirmed analytically by substituting the above parameter values into the following Fershtman and Judd (1987) equilibrium expressions, extended by Rothschild for differentiated duopoly:

\[
\lambda_1 = \lambda_2 = \frac{2bc(2b + d) - ad^2}{c(4b^2 + 2bd - d^2)} = 0.888
\]

for the equilibrium incentive factor,

\[
q_1 = q_2 = \frac{2b(a - c)}{4b^2 + 2bd - d^2} = 71579
\]

for equilibrium quantity,

\[
p_1 = p_2 = \frac{a(2b^2 - d^2) + 2bc(b + d)}{4b^2 + 2bd - d^2} = 14.26
\]

for equilibrium price, and

\[
\Pi_1 = \Pi_2 = \frac{2b(a - c)^2 (2b^2 - d^2)}{(4b^2 + 2bd - d^2)^2} = 448310
\]
for equilibrium gross profit. Applying tax at 40% confirms the above figure of €268,986 for net profit.

I now introduce a cost asymmetry into the simulation model by reducing unit variable cost for firm two from €8 to €7. As expected the firm with lower costs outperforms the firm with higher costs as shown in figure 2. Profit for firm two is higher than for the symmetric case while that for firm one is lower. Incentive factors are also asymmetric: firm two’s incentive factor increased to 0.891 whereas firm one’s has reduced to 0.754. Owners of the lower cost firm are offering greater incentive to managers to maximise sales. Quantity selected by each firm is asymmetric with 70,048 placed on the market by firm one and 77,321 placed on the market by firm two. Unit prices are also asymmetric and have reduced for both firms to 14.13 and 13.77 for firms one and two respectively. Note that the lower cost firm places a greater quantity on the market and achieves a lower price than for the symmetric case whereas the higher cost firm places a lower quantity on the market and achieves a higher price than before.

![Figure 2. Differentiated incentivised Cournot duopoly with asymmetric costs: \( \phi_1 = \phi_2 = 0; c_1 = 8, c_2 = 7 \)](image)
I now introduce advertising into the duopoly model. Borden (1947) suggests that advertising shifts the demand curve to the right or tilts the demand curve or both. Friedman (1983) suggests that the impact of advertising is cumulative over time and that interaction of advertising may occur i.e. advertising by one firm may benefit another firm. This model meets both of Friedman’s conditions. I assume that advertising shifts the demand function to the right and I express this using a simple linear model:

\[ a_i = a_i^* + \phi_i A_i + \rho \phi_j A_j \quad i = 1,2; j = 3 - i \]

where \( a_i \) is the reservation price after advertising, \( a_i^* \) is the reservation price before advertising, \( \phi_i \) is the advertising response of one firm, \( \phi_j \) the advertising response of its rival, and \( \rho \) is the advertising interaction factor representing the relative response of one firm to another firm’s advertising. Where \( \rho \) is positive advertising is cooperative i.e. advertising by one firm also assists the other firm; where \( \rho \) is negative advertising is predatory i.e. gain in demand from advertising by one firm is at the expense of the other firm. Where \( \rho \) is zero then advertising by one firm has no effect upon the other firm. Note that where \( \phi \) is the own advertising effect, \( \rho \phi \) is the cross advertising effect, i.e. the effect of one firm’s advertising on the other firm. Parameters \( A_i \) and \( A_j \) represent the amounts of advertising selected by the firm and its rival respectively. The simulation model uses the Dorfman Steiner (1954) condition to select the optimal amount of advertising for each firm. The amount of advertising represents a cost to the firm and is subtracted from gross profit.
Figure 3. Differentiated incentivised Cournot duopoly with symmetric advertising and symmetric costs: $\phi_1 = \phi_2 = 0.000013; c_1 = c_2 = 8$.

I will firstly examine the scenario when both firms advertise but where advertising effectiveness is low. Advertising effectiveness for both firms is set at 0.000013 which means that €10000 worth of advertising will increase the value of the reservation price by €0.13. This shifts the demand function to the right. The advertising interaction parameter is set at zero and so no spillover of advertising from one firm to the other occurs. Results for this scenario are shown in figure 3 where it is clear that both firms come to a terminal Cournot equilibrium and little advertising is selected.
As levels of advertising effectiveness decrease below 0.000013 behaviour is similar to that shown in figure 3 but the rate of growth is slower.

Figure 4. Differentiated incentivised Cournot duopoly with symmetric advertising and symmetric costs: $\phi_1 = \phi_2 = 0.000015; c_1 = c_2 = 8$.

I now increase the effectiveness of advertising for both firms to 0.000015. Results for this scenario are shown in figure 4. Larger amounts of advertising are selected causing the demand function to shift to the right. In contrast to the previous scenario demand, and consequently revenues and profits, increase exponentially over time and does not come to equilibrium. As levels of advertising effectiveness increase above
0.000014 behaviour is similar to that of figure 4 but growth is more rapid. Clearly a bifurcation in duopoly behaviour has occurred: for advertising effectiveness at or below 0.000013 firms tend to an equilibrium above the standard Cournot level; for advertising effectiveness at or above 0.000014 both firms achieve exponential growth.

Figure 5. Differentiated incentivised Cournot duopoly with asymmetric advertising and symmetric costs: $\phi_1 = 0.000013$, $\phi_2 = 0.000012$; $c_1 = c_2 = 8$.

I now introduce an asymmetry into the duopoly model. I return to the scenario where advertising effectiveness was low leaving firm one’s effectiveness at 0.000013 and reducing that for firm two to 0.000012. Results for this scenario are given in figure 5.
and show that creating an advertising asymmetry has resulted in asymmetric performance: firm one advertises more heavily than and gains a competitive advantage over firm two. Both firms still tend asymptotically to a terminal Cournot equilibrium but firm one comes achieves a higher level of profit than does firm two.

![Graph](image1)

(a)

![Graph](image2)

(b)

**Figure 6.** Differentiated incentivised Cournot duopoly with asymmetric advertising and asymmetric costs: \( \varphi_1 = 0.000013, \varphi_2 = 0.000012; c_1 = 8, c_2 = 7. \)

I now introduce asymmetric costs in addition to asymmetric advertising by reducing unit cost for firm two from €8 to €7. Results in figure 6 show that this leads to a switch in industry leadership: firm two now outperforms firm one. If we regard the
lower cost firm as adopting a cost leadership strategy and the firm with greater advertising effectiveness as adopting a differentiation strategy then we see here that the cost leader outperforms the differentiator. Trial and error experimentation shows that cost leadership outperforms a differentiation strategy provided that unit cost of the cost leader lies at or below €7.8. Figure 7 shows results where unit cost of the cost leader is €7.9. Initially the cost leader outperforms the differentiator but after a time a switch in industry leadership takes place and the differentiator outperforms the cost leader.

![Figure 7. Differentiated incentivised Cournot duopoly with asymmetric advertising and asymmetric costs: $\phi_1 = 0.000013$, $\phi_2 = 0.000012$; $c_1 = 8$, $c_2 = 7.9$](image)

I now introduce asymmetry to the scenario where advertising effectiveness is high. I reduce the advertising effectiveness of firm two from 0.000015 to 0.000014. This dramatically changes the behaviour of the duopoly as shown in figure 8. The more effective advertiser selects larger amounts of advertising stimulating demand for its product and after a time drives its rival out of the market. A small change in advertising effectiveness can dramatically raise or lower firm performance. This
suggests that when costs are identical it is worthwhile for a firm to improve the effectiveness of its advertising.

![Diagram](image1)

(a)

![Diagram](image2)

(b)

**Figure 8. Differentiated incentivised Cournot duopoly with asymmetric advertising and symmetric costs: φ₁= 0.000015, φ₂=0.000014; c₁=c₂=8**

I now introduce asymmetric costs in tandem with asymmetric advertising. Figure 9 shows results for this scenario. Note that a change in industry leadership has taken place: the lower cost firm from the beginning outperforms the more effective advertiser and eventually drives it out of the market. Trial and error experimentation showed that for this level of advertising asymmetry when unit cost for firm two is at or below €7.2 then the lower cost producer achieves industry dominance whereas
above that figure a switch in industry leadership takes place with the more effective advertiser eventually dominating the industry.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9a.png}
\caption{(a) Time Retained_earnings}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9b.png}
\caption{(b) Time Advertising}
\end{figure}

\textbf{Figure 9.} Differentiated incentivised Cournot duopoly with asymmetric advertising and asymmetric costs: $\phi_1 = 0.000015$, $\phi_2 = 0.000014$; $c_1 = 8$, $c_2 = 7$

I now increase the level of advertising asymmetry. I reduce advertising effectiveness for firm two from 0.000014 to 0.000013. This changes significantly duopoly behaviour as shown in figure 10. Firm two’s initial advantage due to low cost is eroded over time until eventually firm one’s advertising advantage predominates. Firm two is eventually driven from the marketplace. This result implies that a cost
advantage will only carry a firm so far. If the advertising asymmetry is too great then
the stronger advertiser will eventually dominate. Figures 8 and 9 demonstrate the
reverse: an advertising advantage alone will only carry a firm so far; if cost
asymmetry becomes too great then the lower cost firm will win out.

Figure 10. Differentiated incentivised Cournot duopoly with asymmetric
advertising and asymmetric costs: $\varphi_1 = 0.000015$, $\varphi_2 = 0.000013$; $c_1 = 8$, $c_2 = 7$

CONCLUSION

This paper examined the behaviour of two firms intercompeting under incentive
Cournot conditions ie. a two stage game is played where owners select incentive
factors for managers in the first stage, and managers select quantity at the second or
market stage. The examination was carried out using a computer simulation model
founded on industrial organisation economic theory. A particular scenario examined
was that where one firm adopted a cost leadership strategy and the other adopted a
differentiation strategy where that differentiation was achieved through advertising.

Several specific results were determined. The simulations demonstrated the existence
of a bifurcation point above which firms advertise relatively heavily and greatly
benefit from that advertising and below which firms advertise little with little consequent change in demand. The bifurcation point is a function of advertising effectiveness. This finding suggests an explanation for the empirical observation that some industries advertise and some do not.

In general the results demonstrate that the impact of advertising is uncertain and unpredictable, even though the models used are deterministic. Markedly different duopoly behaviour was demonstrated when advertising effectiveness crossed threshold levels or became asymmetric. The results suggest that firms cannot be certain about the impact of their advertising policies. A small change in behaviour by the rival firm may lead to very different results for the firm. For example, if the rival firm improves the effectiveness of its advertising the model demonstrates that firm performance may switch from exponential growth to exponential decay.

The results also demonstrate that differentiation by itself does not necessarily confer an advantage on one firm or the other. Under Dixit’s second form of differentiation (I will label this symmetric differentiation) both firms behave symmetrically and achieve equal profits for all time. It is only when an asymmetry is introduced into the model that one firm gains a competitive advantage over the other. Advertising is one mechanism for introducing demand growth into the model. However, while advertising may improve the profit position of both firms, as long as the model remains symmetric no competitive advantage will be conferred on one firm or the other by advertising. Asymmetry may be introduced in a number of ways: by varying advertising effectiveness, cost, managerial incentive, or a combination of these
parameters. Asymmetry is also introduced by nature; this may be simulated by introducing a random element to the impact of advertising.

The results also shed light on the theory of competitive strategy. Whether a low cost or a differentiation strategy is dominant is shown to be contingent on circumstances. Porter (1980, 1985) advises firms that when adopting a strategy based on one attribute that firms maintain close proximity in other attributes. The simulations bear this out. When adopting a low cost strategy firms must maintain a minimum level of advertising effectiveness to remain competitive, or when adopting an advertising effectiveness advantage firms must maintain proximity in costs. The paper suggest a role for management: to identify sources of asymmetry and to introduce such asymmetry into the competitive environment.

**BIBLIOGRAPHY**


Friedman, James (1983). ‘Advertising and oligopolistic equilibrium’, *Bell Journal of 


Rothschild, R. (2002). Discussion of unpublished working paper with author, 
University of Lancaster.