Competition under Bertrand duopoly: low cost versus differentiation

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Abstract

This paper proposes a model of optimal advertising for a duopoly. The advertising model is based on Dorfman and Steiner’s (1954) optimality condition. The duopoly model is as detailed by Dixit (1979) which in turn is based on Cournot (1838) and Bertrand’s (1883) original work. The paper develops the model analytically and then examines the behaviour of the firms in a duopoly using computer simulation. The paper examines the particular case where one firm adopts a low cost generic strategy and the other firm adopts a differentiation strategy where differentiation is achieved through advertising (Porter, 1980 and 1985). The paper finds that there exists a threshold level of advertising effectiveness above which the differentiator wins out and below which the low cost operator wins out. The paper also suggests that the ‘best cost’ strategy, where the low cost operator achieves proximity in differentiation, results in the firm gaining a sustainable competitive advantage.

Introduction

This paper examines the behaviour of two firms in a duopoly where price is the market decision variable and where each of the two firms adopts a different competitive strategy (Porter, 1980, 1985): one firm acts as a low cost operator while the other firm differentiates its product. Kaldor (1950) suggests that advertising may be used to differentiate one firm’s product from another in the eyes of the consumer. Borden (1947:162) suggests that advertising shifts the demand function to the right or
alters the slope of the demand function. In this paper I assume that advertising differentiates a product by shifting its demand function to the right but does not alter its slope.

This examination uses the Dixit (1979) model to simulate differentiated duopolistic competition. The model assumes that each firm faces a downward sloping demand function and that both firm prices influence the quantity selected for each firm. The products are not perfect substitutes and so firm quantity is influenced more by its own price than by that of its rival. The Bertrand equilibrium point represents a pair of prices such that neither firm can increase profit by unilaterally changing its price (Besanko et al, 2003:226). It is this interdependence of firm decision making that makes competition under duopoly uniquely different to either monopoly or perfect competition.

The choice of the optimal amount of advertising is made according to the Dorfman-Steiner (1954) condition. This condition is expanded by Martin (2002:277) who states that the optimal amount of advertising is proportional to revenue, the proportion being the ratio of advertising elasticity of demand over price elasticity of demand. This is a specific instance of the common rule of thumb whereby advertising expenditure is a percentage of revenue. Advertising then increases the intercept (reservation price) of the firm’s demand function linearly in proportion to an advertising response parameter. Of course there is a cost that a firm that advertises must bear: firm profits are reduced by the amount of advertising.

Duopoly theory assumes that both firms make a market decision that achieves an equilibrium such that neither firm, given the decision of the other firm, can increase profit by unilaterally changing its decision (Cournot, 1838). Several authors have recognised that owners and managers are differently motivated with owners interested in maximising profits and managers interested in maximising sales (Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987). The Fershtman and Judd (1987) model solves duopoly as a two stage game where in the first stage owners determine an optimal incentive to apply to managers and in the second stage managers, taking into account this incentive, make the market decision. Therefore under Bertrand conditions managers will select price and the market will then determine the pair of quantities sold at that pair of prices. I use Fershtman and Judd’s (1987) method as the basis for the simulation.

**The model**

Competition in a Bertrand duopoly was examined using a computer simulation model based on the work of Fershtman and Judd (1987) with the addition of the Dorfman-Steiner (1954) condition to determine optimal advertising. Initial conditions for both firms are assumed to be identical except that firm one’s generic strategy is to differentiate its product by means of advertising whereas firm two’s generic strategy is to produce at lower cost than its rival. To operationalise these different strategies two parameters – advertising response and unit cost - are set to different values for the two firms. In each scenario examined firm one, which acts as the differentiator, is given a value for its response to advertising parameter; advertising response for the low cost firm was assumed to be zero ie. the low cost firm does not advertise. Zero
advertising interaction is assumed: the demand function of the low cost firm is not altered by the advertising of the differentiator. For most of the scenarios examined the variable cost for the low cost firm was set at £7 per unit produced and the variable cost for the differentiator was set slightly higher at £8. Values of all other parameters are identical for both firms. Both firms faced the same inverse demand function initially: the value of the intercept (reservation price) was 25, the slope in direction of own price was set at 0.0001, and the slope in direction of rival firm price was set at 0.00005. This means that price for the product decreases by £1 for every additional 10,000 units of the product placed on the market and decreases by £0.50 for every additional 10,000 units of the rival’s product placed on the market. Advertising response values used are of the order of 0.000015 which means that £10,000 worth of advertising will increase the value of the reservation price (intercept of the demand function) by £0.15. Note that own-price and cross-price parameters are different; therefore products are not perfect substitutes. This must be the case when modeling competition under Bertrand conditions as under perfect substitution the model becomes degenerate and unsolvable. Natural growth and decay was suppressed and so any shift in the demand function was due strictly to advertising.

The mechanism used to differentiate one product from another over time was to shift the demand function for the differentiator to the right; in practice this required increasing the value of the intercept (reservation price) as a result of advertising. The greater the amount of advertising the greater the shift to the right. The slope of the demand function does not alter over the course of the simulation. The non-advertising firm’s demand function stayed constant throughout the simulation. As the two demand functions move away from each other, while all the time remaining parallel, the two products become more and more differentiated.

As noted by Dixit (1979) two different interpretations of differentiation are taking place here. Firstly, products are not perfect substitutes and so are differentiated in that respect; this is modeled using different values for own price and cross price parameters (but note that the same values are used for the two firms). In three dimensions the demand surfaces for the two firms are identical, but the slope of the demand surface is steeper in the direction of own firm quantity than it is in the direction of rival firm quantity; for this simulation work it has been set exactly twice as steep. Differentiation in this respect does not change over the course of the simulation. Also here one cannot say which of the two firms is the differentiator: both firms are equally different to each other.

The second interpretation of differentiation is due to the two demand curves moving apart as a result of advertising; the amount of differentiation in this respect does change over time and one can say that one of the two firms – the advertiser – is actively pursuing a differentiation strategy ie. actively setting the two demand functions further apart over time. Dixit (1979) demonstrates that these two aspects of differentiation can have opposing influences on a competitive situation.

**Incentive duopoly competition**

I will examine the duopoly situation where the owners of the two competing firms delegate responsibility for the market decision to managers (Fershtman and Judd, 1987). In the first stage owners optimally select the incentive factor that will apply in
the second stage knowing that managers will optimally select price in the second stage. The simulations were carried out using a simulation model developed by the author and executed using a simulation software package with parameter values as given above. I will start with the symmetric situation where both firms have unit variable cost of £8 and neither advertises. This gives the symmetric result for the two firms shown in figure 1 where both firms earn profit after tax of £267,504 producing a quantity of 72,121 units each period and selling them at a price of £14.18 per unit. These are the outcomes at the Bertrand equilibrium and they hold for all time.

![Figure 1](image1.png)

**Figure 1. Differentiated Bertrand duopoly with symmetric firms (c₁=c₂ = 8)**

I will extend the base case to include a cost asymmetry (c₁ = £8; c₂ = £7) between the two firms but continue restricting both from advertising. The Bertrand equilibrium results in a lower price, higher quantity, and higher profit for the lower cost firm and this equilibrium holds for all time. Results in figure 2 show clearly that the firm with lower costs outperforms the higher cost rival for all time.

![Figure 2](image2.png)

**Figure 2. Differentiated Bertrand duopoly with cost asymmetry (c₁=8, c₂=7, n=30)**

I now extend the base case once again, this time to allow firm one to advertise while keeping unit variable costs symmetric at £8. I therefore set the value of the advertising response parameter for firm one, Φ₁, equal to 0.000013. This means that £10000 worth of advertising will increase the reservation price by £0.13. Results for this are given in figure 3 and show clearly that both firms remain in the market with
the differentiator achieving a sustainable competitive advantage over its rival. Note that this is achieved by a remarkably low amount of advertising.

\[
\begin{align*}
\text{Time} & \quad \text{Retained earnings} \\
0 & \quad 20 & \quad 40 & \quad 60 & \quad 80 & \quad 100 \\
1 & \quad 2 & \quad 2 & \quad 1 & \quad 1 & \quad 1 & \quad 1
\end{align*}
\]

(a)

\[
\begin{align*}
\text{Time} & \quad \text{Advertising} \\
0 & \quad 20 & \quad 40 & \quad 60 & \quad 80 & \quad 100 \\
1 & \quad 2 & \quad 2 & \quad 1 & \quad 1 & \quad 1 & \quad 1
\end{align*}
\]

(b)

Figure 3. Differentiated Bertrand duopoly with advertising asymmetry (\(c_1 = c_2 = 8, \Phi_1 = 0.000013, \Phi_2 = 0\))

I will now formulate the main results being reported in this paper as a proposition.

**Proposition.**

In a differentiated Bertrand duopoly where optimal incentive factors are selected by owners for managers and where one firm adopts a differentiation strategy using advertising and the other adopts a low cost strategy and does not advertise then

a) there exists a threshold level of advertising effectiveness above which the differentiator outperforms the low cost operator and below which the low cost operator outperforms the differentiator

b) for a given level of advertising effectiveness there exists a threshold level of cost asymmetry above which the low cost operator outperforms the differentiator and below which the differentiator outperforms the low cost operator

c) the introduction of a significant random component to the impact of advertising did not alter these results

I now set the unit variable cost of the low cost operator to £7 and keep the cost of the differentiator at £8. I allow the higher cost firm to differentiate its product by advertising. I set the differentiator’s advertising response to 0.000015 which means that £10000 worth of advertising increases the reservation price by £0.15; this in turn causes the entire demand function for the advertiser to shift to the right. I do not
allow the low cost operator to advertise i.e. its advertising response remains at zero. Figure 4a shows clearly that the differentiator quickly overtakes and outperforms the low cost operator. Figure 4b shows that when advertising response is high the differentiator selects large amounts of advertising shifting its demand function significantly. This results in the differentiator increasing the quantity of its product demanded while decreasing the quantity demanded of its rival. Ultimately (in period 34) the low cost rival is driven out of the market.

![Graph A](image)

**Figure 4a.**

**Figure 4.** Differentiated Bertrand duopoly with high advertising asymmetry and low cost asymmetry ($c_1=8$, $c_2=7$, $\Phi_1=0.000015$, $\Phi_2=0$).

The above scenario assumed that the low cost firm did not advertise. However the results hold even if the low cost firm advertises provided its response to advertising is below 0.000014. If advertising response is at or above this level then the low cost operator will gain the competitive advantage as shown in figure 5. This combination of lower cost and proximity in advertising response is difficult to beat and broadly corresponds to the category of ‘best cost’ recently added to Porter’s taxonomy of generic strategies (Thompson and Strickland, 2003:167). Note that the kink in retained earnings for firm two, evident in figure 5, is due to relatively high advertising expenditure in periods 28 and 29 causing a temporary reduction in profits.

![Graph B](image)
Figure 5. Differentiated Bertrand duopoly with low advertising asymmetry and low cost asymmetry (c₁ = 8, c₂ = 7, Φ₁ = 0.000015, Φ₂ = 0.000014).

A small reduction in the effectiveness of advertising dramatically alters the situation examined in figure 4. The simulation was carried out again this time with the advertising effectiveness factor for the differentiator reduced from 0.000015 to 0.000013, a reduction of 13%, and the advertising response for the low cost operator reset to zero. All other parameters remained unchanged. The results of this simulation in figure 6 show that the low cost operator outperforms the differentiator at the outset and continues to do so for all time. Due to lower advertising response only a small amount of advertising is selected as shown in figure 6b; this contrasts with the much greater amount of advertising shown in figure 4b. Also in contrast to the previous situation both firms remain in the market for all time. Note also the difference between duopoly behaviour here and that of figure 3 where both firms had equal costs.
Figure 6. Differentiated Bertrand duopoly with low advertising asymmetry and low cost asymmetry ($c_1 = 8$, $c_2 = 7$, $\Phi_1 = 0.000013$, $\Phi_2 = 0$).

Trial and error experimentation showed that the value 0.000013 for advertising response is a change point for this cost regime. At or below this threshold level the low cost operator will sustain a competitive advantage; above this value the differentiator will sustain a competitive advantage.

b) I will now explore the impact of an increase in cost asymmetry. When unit variable cost of the differentiator was increased from £8 to £10 and all other variables remained at their initial values behaviour of the duopoly altered significantly as shown in figure 7. Here the low cost operator outperforms the differentiator for all time. Although advertising response for the differentiator is at its original level of 0.000015 figure 7b shows that the amount of advertising selected by the optimiser is much less that that shown in figure 4b. However if the differentiator can increase its response to advertising by one significant digit to 0.000016 then it increases advertising expenditure and regains the competitive advantage. For the differentiator to retain its competitive advantage an increase in its cost requires a corresponding increase in its response to advertising.
Figure 7. Differentiated Bertrand duopoly where advertising asymmetry is high and cost asymmetry is high (c1= 10, c2=7, Φ1= 0.000015, Φ2= 0).

Now I will examine the scenario where an increase in cost asymmetry is brought about by a reduction in unit cost of the low cost operator rather than by an increase in cost of the differentiator. I find that unit cost of the low cost operator must go down to 1 before the low cost operator can outperform the differentiator. This is quite a decrease and implies that a low cost operator strategy is unlikely to outcompete a differentiation strategy where advertising response of the differentiator is high. If advertising response of the differentiator is lowered to 0.0000134, just above the threshold level, then the unit cost of the low cost operator need only reduce from 7 to 6.7 for the low cost operator to gain the strategic advantage. Therefore the closer is the advertising response to the threshold level the less cost asymmetry is needed for the low cost operator to outperform the differentiator.

c) The above simulations assume that the impact of advertising is determinate and in proportion to the advertising response parameter. In this section I will assume that the impact of advertising has a random component: I will assume that the impact of advertising is plus or minus 30% of its determinate value. I re-examined the above scenarios under with this random component to advertising and found the above threshold levels to be robust. While details of the behaviour altered somewhat the type of behaviour and the different zones of behaviour held. Where costs of the differentiator and low cost operator were £8 and £7 respectively and where the low cost operator did not advertise the threshold level of advertising response was again found to be 0.0000133. Above this level the differentiator outperformed the low cost operator; at or below this level the low cost operator outperformed the differentiator.

**Conclusion**

This research project examined the behaviour of two firms intercompeting where one firm adopted a low cost generic strategy and the other adopted a differentiation strategy and where differentiation was achieved through advertising. Both firms competed under incentive Bertrand conditions i.e. owners selected incentive factors for managers in the first stage, and managers selected price as the decision variable at the second or market stage. The examination was carried out using a computer simulation model founded on industrial organisation economic theory. A contribution of the paper is to extend the Fershtman and Judd (1987) incentive duopoly model to
include optimal advertising according to the Dorfman-Steiner (1954) condition. This creates an additional feedback loop and allows interesting duopoly behaviour to be modeled.

The results show that neither a low cost nor a differentiation strategy automatically yields a competitive advantage. The outcome depends on the combination of cost and advertising asymmetries: sometimes the differentiator outperforms the low cost operator and at other times the reverse happens. Several specific results were found.

Firstly, where the low cost operator chooses not to advertise at all, trial and error experimentation determined the existence of a threshold level of advertising effectiveness for the differentiator above which the differentiator gained the competitive advantage and below which the low cost operator gained the advantage. If a firm can achieve a level of advertising effectiveness above the threshold level, either due to increasing its own advertising capability or because the market responds well to advertising, then it will always pay to advertise provided the rival adopts a pure low cost strategy.

Secondly, for a given advertising asymmetry threshold levels of cost asymmetry were shown to exist above which the low cost operator gained competitive advantage and below which the differentiator gained advantage. The implication for firms is that an advertising advantage only carries a firm so far; the firm must also keep its costs in line in order to sustain a competitive advantage.

Thirdly, the research suggests that a ‘best cost’ strategy - where the low cost operator also achieves proximity in advertising effectiveness – is particularly effective and can yield significant competitive advantage.

Fourthly, the findings were robust even on introduction of a significant random component to the impact of advertising.

Finally the research shows the usefulness of a simple simulation model as a vehicle for examining strategic initiatives experimentally.

**Bibliography**


