The incentive model of the firm: an analytical and simulation approach

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Abstract

In keeping with the theme of this conference this paper examines the boundary between owners and managers and the different motivations that owners and managers have in respect of the performance of the firm. In particular the paper examines the use of the incentive model proposed by Fershtman and Judd (1987) as an objective function for managers. This objective function aims to maximise a linear combination of profit and sales in contrast with the standard neoclassical model which aims to maximise profits alone. The paper firstly examines this model from an analytical point of view and then examines its dynamic behaviour by simulating the model using computer support. The paper seeks to identify if there is an optimal incentive factor for owners to choose when setting the objective function for managers.

This work is part of the author's PhD research and represents ongoing rather than completed work.
Introduction

This examination of the theory of the firm takes as its starting point the managerialist model of Fershtman and Judd (1987). This model assumes that the objectives of owners and managers are different: managers are assumed to be motivated both by the amount of profits and by the amount of revenues; owners on the contrary are assumed to be motivated by the amount of profits only. This model specifically takes into account the fact that the amount of sales matters to managers as they perceive that their income, perquisites and status is influenced more by amount of sales than amount of profits.

Fershtman and Judd used their incentive model to examine the behaviour of firms in a duopoly and concluded that their incentive model leads to a Cournot equilibrium that is more efficient than the standard Cournot equilibrium based on profit maximising in that the combined output of the two firms is greater than at standard Cournot equilibrium, price is lower, and more of the output is produced by the lower cost firm.

This paper examines the incentive function as an alternative to the standard profit maximisation function in the author's simulation model of the growth of the firm already documented in Brady (1999, 2000, 2001a and b).

Analysis

The Fershtman and Judd model expresses the objective function of managers as follows:

\[ O = \alpha \Pi + (1-\alpha)R \]  \hspace{1cm} \ldots (1)

where \( O \) is the managerial objective function, \( \Pi \) represents profit, \( R \) represents sales revenue, and \( \alpha \) and \( 1-\alpha \) represent the weighting attributed to profits and sales respectively. Managers use this objective function as a basis for setting price and determining output quantity. Owners are motivated to maximise profits and attempt to control the behaviour of managers by specifying the value of the incentive parameter \( \alpha \). Managers are assumed to act so as to maximise \( O \) as their reward is assumed to be a function of \( O \):

\[ I = A + B*O \]

To determine the incentive maximising behaviour of managers we begin by substituting revenue minus cost for profit \( \Pi \) in equation 1 above to get:

\[ O = R - \alpha C \]

This objective function will have the effect of making managers less sensitive to cost as not all costs are set against revenue but only a proportion of the costs. Now, following standard procedures of calculus we differentiate the objective function with respect to quantity:
\[ \frac{dO}{dq} = \frac{d}{dq} (R - \alpha C) \]

\[ = \frac{dR}{dq} - \alpha \frac{dC}{dq} \]

Setting this to zero to determine the turning point (maximum or minimum) and substituting MR for \( \frac{dR}{dq} \) and MC for \( \frac{dC}{dq} \) we get:

\[ MR = \alpha MC \quad \ldots \quad (2) \]

This equation is the standard marginalist equation with the inclusion of \( \alpha \), the managerial incentive factor. When \( \alpha \) is equal to one the objective function for managers reduces to \( O = \Pi \) and equation 2 above becomes the standard marginalist equation \( MR = MC \).

The incentive model thus leads to a greater level of output and a lower price at equilibrium than does the profit maximisation model. This is no surprise as we have already determined that incentivised managers will be less sensitive to rising costs. Figure 1 shows profit maximising equilibrium occurring at point X. Incentive equilibrium will occur at a point to the right of X, ie. at a greater level of output, for values of \( \alpha \) between 0 and 1. In this example incentive equilibrium occurs at point a, giving price p and output q. The incentive factor \( \alpha \) is equal to \( \frac{aq}{ab} \).

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**Figure 1: Incentive model equilibrium point**

Fershtman and Judd (1987) use a linear demand function and a constant unit cost function in their model. The analysis becomes more difficult as more complicated functions are used to represent demand and costs. Let us examine the incentive model when the cost function is a quadratic ie. diseconomies of scale exist.

\[ P = a - bQ \]

\[ C = cQ^2 \]

\[ R = Q (a - bQ) \]
\[ \Pi = R - C \]
\[ = Q(a - bQ) - cQ^2 \]
\[ = aQ - (b + c)Q^2 \] \hspace{1cm} \ldots (3)

For profit maximisation we differentiate profit \( \Pi \) with respect to quantity \( Q \), set the result to zero, and solve for quantity \( Q \) as follows:

\[ \frac{d\Pi}{dQ} = a - 2(b+c)Q = 0 \]

\[ \Rightarrow Q = \frac{a}{2b + 2c} \]

Substituting the above expression for optimal \( Q \) in the profit function given in equation 3 we get optimal profit as follows:

\[ \Pi = \frac{a^2}{2b + 2c} - (b + c) \left( \frac{a}{2b + 2c} \right)^2 \]

\[ = \frac{a^2}{4(b + c)} \]

Now let us examine the incentive model and see how it implies different behaviour on the part of managers. The incentive model objective function for managers is:

\[ O = \alpha \Pi + (1-\alpha)R \]
\[ = \alpha(aQ - (b + c)Q^2) + (1-\alpha) (Q (a - bQ)) \]

We note that when \( \alpha \) has value one this becomes the profit maximisation model; when \( \alpha \) has a value of zero managers are motivated only by sales and not at all by profits. Again, to determine its maximum value, we differentiate with respect to quantity, set the result to zero, and solve to get:

\[ \frac{dO}{dQ} = \alpha a - 2\alpha (b+c) Q + (1-\alpha) (a - 2bQ) = 0 \]

\[ \Rightarrow Q = \frac{a}{2b + 2\alpha c} \]

For values of \( \alpha \) less than 1 the optimal quantity under the incentive model is greater than under the profit maximisation model. Note that we could have used \( MR = \alpha MC \) (equation 2) instead to determine the optimal value of \( Q \).

Optimal amount of profit can then be determined by substituting the above term for \( Q \) in equation 3:

\[ \Pi = a( \frac{a}{2b + 2\alpha c}) - (b + c) \left( \frac{a}{2b + 2\alpha c} \right)^2 \]
\[ = a^2 (b + (2\alpha - 1) c) / 4(b + \alpha c)^2 \]

To determine the value of the incentive factor \( \alpha \) for which profit is a maximum we differentiate profit with respect to \( \alpha \), set the result to zero and solve to get:

\[ \frac{d\Pi}{d\alpha} = \frac{2a^2c^2 (1 - \alpha)}{4(\alpha c + b)^3} = 0 \]

\[ \Rightarrow \alpha = 1 \]

This is no surprise as when the value of \( \alpha \) is one we have the standard profit maximisation model.

The situation being examined here is that of monopolistic competition where decisions of firms are independent. Fershtman and Judd examine the case of firms in a duopoly where quantity and price setting decisions of firms are interdependent. They conclude that in a Cournot equilibrium owners of firm A will be motivated to set \( \alpha \) for their manager to a value less than one as by doing so they can influence the behaviour of the manager of firm B in a direction that suits owners of firm A.

This research paper does not examine the interdependent behaviour of firms in an oligopoly but leaves that for a later day. It focuses instead on examining the growth of firms when managers are given the Fersthman and Judd incentive function and the cost function is more complex than constant variable cost. One advantage of a simulation approach is that more complex functions for demand and cost can be used and the model is still ‘solvable’, albeit for specific values of variables rather than in a general form. The author's simulation model (Brady, 2000) uses a cubic function to model the S-shaped behaviour of firm costs as quantity increases. To simulate the behaviour of the firm under the incentive model we need only include the incentive factor \( \alpha \) in the simulation model. Using the linear demand function and cubic cost function from that model we can determine output quantity as follows:

\[ MR = a - 2bQ \]
\[ MC = d - 2eQ + 3fQ^2 \]
\[ MR = \alpha MC \]

\[ \Rightarrow a - 2bQ = \alpha (d - 2eQ + 3fQ^2) \]

\[ \Rightarrow Q = \frac{((2\alpha e - 2b) + \sqrt{(2b - 2\alpha e)^2 - 12\alpha f(ad - a))}}{6\alpha f} \] ….. (5)

From equation 5 we can determine optimal output for all demand functions, ie. for all values of parameters a and b. We assume that the cost function and incentive function do not change ie. that parameters d, e and f and parameter \( \alpha \) remain constant. The growth path of the firm under the incentive model is no longer the path of the marginal cost.
curve MC but instead $\alpha MC$, an upward sloping curve similar to but to the right of the marginal cost curve as shown in figure 2.

![Graph showing incentive model growth path]

**Figure 2. Incentive model: growth path**

Now let us examine what happens when we vary the value of alpha. The model was simulated for two hundred time periods and for values of alpha from 0.1 to 1.2. Dividend, depreciation, and interest ratios were set to zero so that the only amount to be taken from profits, other than production costs, is tax (which as the old adage goes is always with us). Note that a value of alpha greater than one implies that managers are penalised for sales; in practice values of alpha greater than one are unlikely to be used and are included here for completeness only.

![Incentive model simulation results graph]

**Figure 3: Incentive model: simulation results**
The results of these simulations are given in figure 3 where price is shown in units, quantity and retained earnings in thousands, and accumulated capital in millions. As demand does not vary during the simulation, the values for price, quantity and retained earnings are constant throughout each period of the simulation; accumulated capital represents the amount of capital accumulated at the end of period 200. From figure 3 we can clearly see that as alpha increases price increases and quantity decreases, which is as expected from the discussion above. We also note that retained earnings and accumulated capital have maximum values when alpha is equal to one; again, this is consistent with the analysis of the quadratic cost function carried out above.

Note that to determine this result analytically for a cubic cost function would have entailed a considerable amount of work. The expression for Q given in equation 5 must be substituted into the profit equation

\[ \Pi = aQ - bQ^2 - (c + dQ - eQ^2 + fQ^3) \]

This expression must then be differentiated with respect to \( \alpha \), the result set to zero, and then solved for \( \alpha \), a non-trivial undertaking.

**Discussion**

We can see that inclusion of the incentive factor \( \alpha \) and the maximisation of a linear combination of profit and revenue makes no material difference to the dynamic behaviour of the firm: the same essential pattern of behaviour over time is observed as in the case of standard profit maximisation. The major difference noted is that under the incentive model the level of output for all values of demand is greater than under the standard profit maximisation model and the price is lower.

![Figure 4. Reduction in retained earnings for different values of alpha.](image-url)
However, to the owners of the firm this difference may still be significant. A plot of profits foregone against values of the managerial incentive factor is shown in figure 4. Values of alpha less than 0.5 do yield substantially lower profits: 30% lower in the case of alpha of value 0.1. However values of alpha closer to one yield much smaller reductions in profits; for example, for an alpha of value 0.7 the reduction in profit is only 3%.

This analysis would seem to support the view that owners can give some reward to managers for increasing sales without significantly jeopardising profits. This would have the effect of appeasing managers who for their own reasons wish to maximise sales.

**Bibliography**


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