Duopoly with strict profit maximiser and strict sales maximiser: examination of influence of cost structures on industry leadership

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Abstract

This paper examines a duopolistic competitive situation where one firm adopts the policy of strict profit maximisation and the other firm the policy of strict sales revenue maximisation and where firms have unequal cost structures. The paper sets out to determine the conditions under which a sales maximiser will outperform a profit maximiser under Cournot Nash equilibrium conditions.

Keywords

DUOPOLY, PROFIT, SALES REVENUE, MAXIMISATION, COST
This paper continues the author’s work on incentive duopoly using the Fershtman and Judd (1987) incentive model of the firm. The approach taken combines analytical and simulation methods: the primary approach is analytical with simulation carried out to confirm the analytical result. This paper reports on analysis that is complementary to but extends previous work of the author (Brady, 2002). In this paper sales maximisation means sales revenue maximisation.

The Fershtman and Judd model of the firm proposes that owners set managers an objective function that is a linear combination of profits and sales:

\[ O = \alpha \Pi + (1 - \alpha) R \]

where \( \alpha \) is the incentive factor and typically has a value between zero and one. A strict profit maximiser uses an incentive factor of value one, and a strict sales maximiser an incentive factor of value zero. This paper considers only the duopoly situation where one firm adopts a strict profit maximisation policy and the other firm adopts a policy of strict sales maximisation.

Previous analysis (Brady, 2002) shows that for an inverse linear demand function \( p = a - bQ \) and linear cost functions for each firm \( C = cq \) we get the following expressions for quantity, profit, and price at the Cournot Nash equilibrium:

\[
q_1 = \frac{1}{3b} \{a - 2c_1\} \\
q_2 = \frac{1}{3b} \{a + c_1\} \\
p = \frac{1}{3} \{a + c_1\} \\
\Pi_1 = \frac{1}{9b} \{a - 2c_1\}^2 \\
\Pi_2 = \frac{1}{9b} \{a + c_1 - 3c_2\} \{a + c_1\}
\]

where subscript one refers to the strict profit maximiser and subscript two to the strict sales maximiser. Previous analysis showed that when costs are equal the sales maximiser will outperform the profit maximiser. In this paper we consider the conditions under which the sales maximiser will outperform the profit maximiser when costs are unequal.

For profits of the sales maximiser to exceed those of the profit maximiser we require

\[ \Pi_2 > \Pi_1. \]

\[ \Rightarrow \{a + c_1 - 3c_2\} \{a + c_1\} > \{a - 2c_1\}^2 \]

\[ \Rightarrow c_2 < \{(2a - c_1) / (a + c_1)\} c_1 \]

\[ \Rightarrow c_2 < \{1 + (a-2c_1) / (a+c_1)\} c_1 \] (1)

Where \( c_2 > c_1 \), variable cost of the sales maximiser can be up to \( \{(a-2c_1) / (a+c_1)\}*100\% \) more than the variable cost of the profit maximiser and still the sales maximiser will outperform the profit maximiser.

Where \( c_2 < c_1 \), then we can show that the above condition holds for \( a > 2c_1 \). We also note, from the expression for \( q_1 \) above, that \( a < 2c_1 \) leads to a negative or infeasible quantity. Therefore, for all feasible quantities (ie. positive quantities) the sales maximiser will outperform the profit maximiser whenever variable cost of the sales maximiser is less than variable cost of the profit maximiser.
Overall we note that costs of the sales maximiser must be substantially higher than those of the profit maximiser before sales maximisation becomes an inferior policy to profit maximisation. For example, where parameter \( a \) has value 25 units and \( c_1 \) has value 6 units, then \( c_2 \) can have value up to 8.51, i.e. 42% greater than \( c_1 \), before sales maximisation becomes less profitable than profit maximisation. Somewhat counterintuitively, as costs of firm one decrease relative to parameter \( a \), the proportion over \( c_1 \) that \( c_2 \) can reach, and firm two still outperform firm one, increases:

<table>
<thead>
<tr>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>proportion above ( c_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>10.2</td>
<td>27%</td>
</tr>
<tr>
<td>6</td>
<td>8.5</td>
<td>42%</td>
</tr>
<tr>
<td>4</td>
<td>6.4</td>
<td>59%</td>
</tr>
<tr>
<td>2</td>
<td>3.56</td>
<td>78%</td>
</tr>
</tbody>
</table>

This would imply that where costs are low relative to prices then sales maximisation may be an appropriate policy for managers to pursue even when the firm’s costs are significantly higher than their competitors.

We can now examine how such a difference in cost structures affects the behaviour of a duopoly over time; to do this we use a computer simulation model developed by the author (Brady, 2001). We will examine two industry growth situations: firstly where demand grows in a logistic (S-shaped) fashion, and secondly where demand is cyclic.

![Diagram](image)

**Figure 1: Duopoly behaviour over time when demand is growing**

In the first case examined, growth in demand is simulated by using a logistic expression to increase the value of demand parameter \( a \) from its initial value of 25. Variable cost for the sales maximiser is 9 and for the profit maximiser is 6. Figure 1 shows how the industry leadership position switches as demand increases. Initially condition (1) above fails and the profit maximiser outperforms the sales maximiser. However as demand increases over time eventually condition (1) is met. This occurs at period 26 when the value of parameter \( a \) reaches 30. At that point the sales maximiser outperforms the profit maximiser and we get a switch in industry leadership.
In the second case we also obtain a switch in industry leadership as shown in figure 2; as change in demand is cyclic we get repeated switching of the leadership position. Note also that the amplitude of the curve for the sales maximiser is greater than that for the profit maximiser, and the difference between the two curves is relatively larger at the trough than at the crest. From a technical point of view, cyclic change in demand is modeled as a sine curve:

\[
d\alpha/dt = k \sin \left\{2\pi (t - \tau) / n\right\}
\]

where \(d\alpha/dt\) represents the rate of change of demand parameter \(\alpha\) with respect to time, \(k\) represents the amplitude of the change, \(\tau\) represents a lag in the starting position of the sine curve (so we can set demand to initially grow or decay depending on the value that we give to \(\tau\)), and \(n\) represents the number of time periods in a single cycle. For this research \(k\) was set to a value of 1, \(n\) to a value of 30, and \(\tau\) to a value of 25 (forcing the cycle to start on the upward sloping portion of the sine curve). As it is not possible to simulate a differential equation on computer, the simulation model uses Euler’s numerical integration method to calculate the values of parameter \(\alpha\) over time:

\[
a_{t+1} = a_t + f(a_t, t) \Delta_t
\]

where \(f(a_t, t)\) is the function represented by the right hand side of the differential equation, \(\Delta_t\) represents the time increment, and \(a_0\) is the initial value of parameter \(\alpha\); an initial value of 25 is used for parameter \(\alpha\) in this research. Cost structures remain as for the first case.
In Figure 3a we see accumulated capital for the two firms. Here we note that accumulated capital for the profit maximiser is greater than that for the sales maximiser in all periods except those between 15 and 18. It is clear in this case that a profit maximisation strategy in the long run outperforms a sales maximisation strategy. However, the profit maximiser cannot be complacent: a very slight downward shift in the sales maximiser’s variable cost from 9 to 8.8 (still 47% higher than the profit maximiser’s cost!) will reverse this situation leaving accumulated capital of the sales maximiser in period 60 greater than that of the profit maximiser (figure 3b).

Two further situations are examined in figures 4 and 5. In figures 4a and 5a we see the results when the value of variable cost for the sales maximiser is reduced to 8. Here the value of parameter a never falls to a level where condition 1 is breeched and so profits for the sales maximiser always exceed those of the profit maximiser. In figures 4b and 5b we see the results when variable cost of the sales maximiser is increased to 10. Here demand parameter a never reaches a value such that condition (1) is met and therefore profits of the profit maximiser always exceed those of the sales maximiser. We also note that the sales maximiser makes actual losses when $a < 3c_2 - c_1$; this occurs when $a < 24$ and in figure 4b this occurs between periods 23 - 29 and periods 52 - 59.
Note that in the model losses are treated separately to capital and recorded as a separate variable. Hence the value of accumulated capital does not reduce when losses are made but instead remains at a constant level as shown in figure 5b. In effect, the assumption is that firms borrow to pay for losses. The penalty that firms incur is that interest is charged on accumulated losses; an interest rate parameter of value 7% is used in the model. This charge accounts for the slight downward trend in the sales maximiser retained earnings curve shown in figure 4b.

![Graphs](image)

**Figure 5. Accumulated capital**

In conclusion we can see that industry leadership depends on the relative cost structures of the two firms in a duopoly and is particularly sensitive to change in demand and to change in the variable cost of the sales maximiser. In any single period variable cost of the sales maximiser can be significantly higher than that of the profit maximiser and still the sales maximiser will outperform the profit maximiser. Looking at multiple periods, where condition (1) always holds the sales maximiser will dominate the duopoly as in figures 4a and 5a. Where condition (1) never holds the profit maximiser will dominate as shown in figures 4b and 5b. When demand fluctuates in such a way that the condition successively holds and is breached then industry leadership switches between the two firms as shown in figures 1 and 2. Domination depends on the exact values of the changing parameters and as shown in figures 3a and 3b a small relative change in the values of the cost parameters can lead to a switch in leadership.

**Bibliography**

